Physics of Neoclassical Tearing Modes

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OUTLINE

- What are NTMs and why are they important?
- Simple physical picture of the instability
- Rutherford model equation
- Survey of experimental observations/ mode characteristics/ implications for ITER
- Rf techniques and other means of stabilization
- Outstanding theoretical and experimental issues.

What are NTMs?

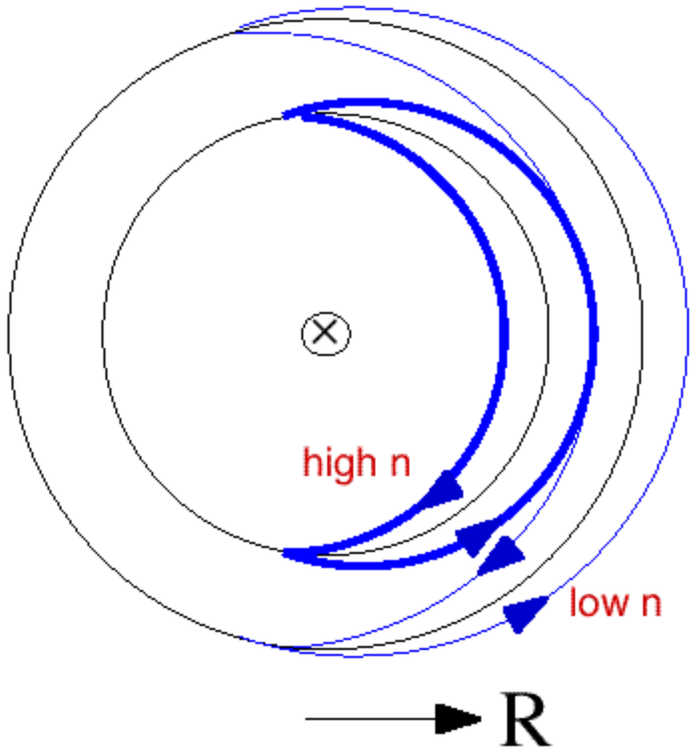
- NTMs are relatively large size **magnetic islands** that develop slowly at mode rational surfaces with low (m,n) mode numbers in **high temperature tokamak** plasmas.

- Like the classical TMs they are current driven but the current source is the **bootstrap current** - a neoclassical (toroidal geometry driven) source of free energy.

- They limit the attainable β in a tokamak to values well below the ideal MHD limit - hence they are a **major concern** for all reactor grade machines i.e. long pulse (steady state) devices.

BOOTSTRAP CURRENT

Projection into a poloidal plane



generated by trapped particles:

example: banana particles

- electrons drift from flux surfaces due to the ∇B-drift
- electrons with low parallel velocity are trapped in the toroidal mirror
  ⇒ **banana orbits**
- at the intersection of 2 banana orbits a net current results due to the density gradient
- passing particles exchange momentum with trapped particles
  ⇒ **bootstrap current**

similar: helically trapped particles

Classical Tearing Modes

- Asymptotic theory- uses two regions of the plasma
  - Outer region - marginal ideal MHD - kink mode
  - Inner region - include effects of inertia, resistivity, nonlinearity, viscosity etc.
- Matching between inner and outer region

$$\boxed{\frac{1}{2}\Delta'\psi_1 = \mu_0 R \int_{-\infty}^{\infty} d\rho \oint \frac{d\alpha}{2\pi}\cos(m\alpha) J_{\parallel},}$$

- Linear theory : $\gamma \sim (\Delta')^{4/5} S^{-3/5}$

Classical TM - contd.

- Near mode rational surface $\mathbf{k}\cdot\mathbf{B} = \mathbf{0}$,

$$B_0 = B(r=r_s) - B_\theta(nq'/m)(r-r_s)\boldsymbol{\alpha} , \boldsymbol{\alpha} = \theta - (n/m)\varsigma$$

$$\delta B = \delta B_r \sin(m\alpha)\, \mathbf{r}$$

- Leads to the formation of a **magnetic island**
- Island width $w = 4(\delta B_r\, r_s / B_\theta\, nq')^{1/2}$
- when w > resonant layer thickness - nonlinear effects important
- Nonlinear Evolution - Rutherford regime

$$\boxed{\frac{dw}{dt} \approx \eta\Delta'} \qquad \boxed{\Rightarrow \; w \; \alpha \; t}$$

- The form of the Rutherford equation can be traced to the form of Ohm's Law which governs the inner region solution, e.g.

$$\boxed{E_{\|} = \eta J_{\|}} \qquad E_{\|} \sim -\frac{\partial A_{\|}}{\partial t} \qquad J_{\|} \sim -\nabla^2 A_{\|}$$

$$\Downarrow \qquad \Downarrow$$

$$\frac{d\delta B}{dt} = \eta \frac{\Delta'}{w} \delta B \qquad \Rightarrow \qquad \boxed{\frac{dw}{dt} \approx \eta \Delta'}$$

- In high temperature tokamaks neoclassical effects need to be retained

Modified Ohm's Law

$$\boxed{< E_{\|} > \;=\; \eta J_{\|} + \frac{1}{neB} < B \cdot \nabla \cdot \pi_{\| e} >}$$

$$\Downarrow$$

Bootstrap current

$$\Uparrow$$

$$\frac{1}{neB} < B \cdot \nabla \cdot \pi_{\| e} > \;\approx\; \frac{\mu_e}{\nu_e} \frac{1}{B_\theta} \frac{dp}{dr} + \eta \frac{\mu_e}{\nu_e} J_{\|}$$

Electron viscous stress which describes damping of poloidal electron flows - new free energy source.

Dependence on pressure gradient, also fraction of trapped particles

Modified Rutherford Equation

$$\boxed{\frac{dw}{dt} = \frac{\eta}{\mu_0}(\Delta' + \frac{D_{nc}}{w})}$$

where $D_{nc} = -\sqrt{\epsilon}\,\frac{2\mu_0}{B_\theta^2} p' \frac{q}{q'} k_0$

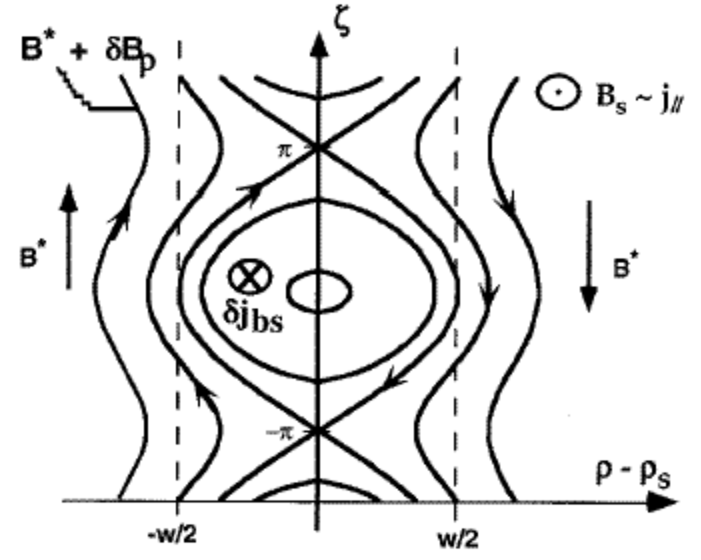
$\boxed{p'q' < 0, \quad D_{nc} > 0}$ **Unstable for normal tokamak operation**

$\boxed{p'q' > 0, \quad D_{nc} < 0}$ **Stable in reversed shear regions**

- Can be unstable for $\Delta' < 0 \Rightarrow \boxed{w_{sat} = \frac{D_{nc}}{-\Delta'} \approx \frac{r_s \beta_\theta}{m}}$
- for small islands $\boxed{w \sim \sqrt{\eta t}}$

PHYSICS OF NTM

- Plasma pressure profile is flattened within the island - J_{bs} **is turned off**
- **This triggers a δJ_{bs} with the same helical pitch as the island**
- the corresponding induced δB has the same direction as the initial perturbation and **enhances it**



This picture neglects finite perpendicular thermal conductivity within the island - important for small island widths - leads to **threshold size**.

Finite perpendicular thermal conductivity effect

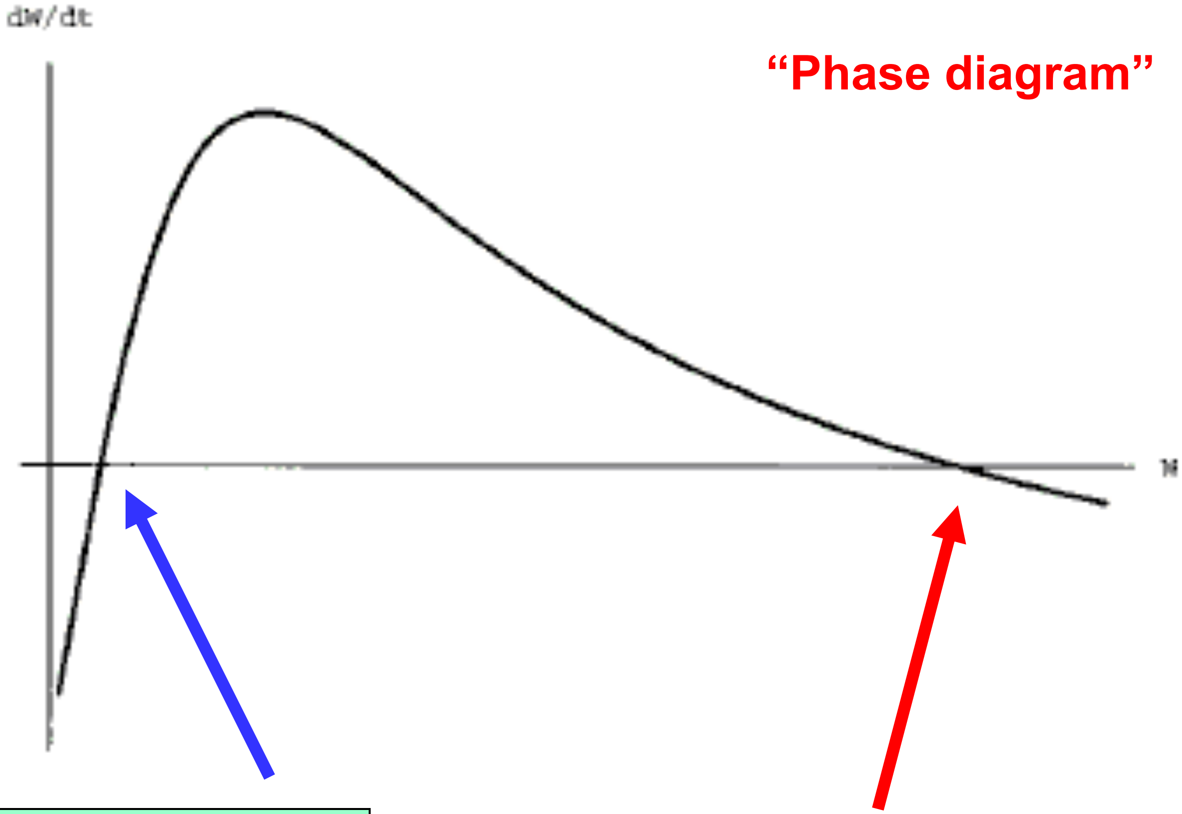
$$\frac{dw}{dt} = \frac{\eta}{\mu_0}(\Delta' + D_{nc}\frac{w}{w^2 + w_c^2})$$

$$w_c \sim \left(\frac{\chi_\perp}{\chi_\parallel}\right)^{1/4} \sqrt{\frac{q^2 R}{mq'}}$$

Threshold - “seed” – island size

$$\boxed{w_{seed} = -\frac{\Delta' w_c^2}{D_{nc}}}$$

NTM characteristics



"Phase diagram"

"seed" island necessary for growth – so NTM is a nonlinear mode "subcritical instability"

⇩

How is the seed island created?

Saturation width proportional to β_θ - hence limits plasma pressure

Two- fluid model generalization + other effects

The density equation,

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{v} = S_n,$$

The momentum equation,

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \equiv \rho[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v}] = \mathbf{j} curl \mathbf{B} - \nabla p - \nabla \cdot \Pi - \nu_\perp \rho \nabla^2 \mathbf{v}.$$

The pressure equation:

$$\frac{\mathrm{dp}}{\mathrm{dt}} = -\frac{5}{3} p \nabla \cdot \mathbf{v} + \frac{2}{3}[\mathbf{Q} - \nabla \cdot \mathbf{q} - \Pi : \nabla \mathbf{v}].$$

The generalized Ohm's law

$$\underbrace{\mathbf{E} + \mathbf{v} \wedge \mathbf{B}}_{ideal\ MHD} = \underbrace{\eta \mathbf{j}}_{resistive\ MHD} + \underbrace{\frac{1}{\epsilon_0 \omega_{pe}^2 (1+\nu)}[\frac{\partial \mathbf{j}}{\partial t} + \nabla ...]}_{electron\ inertia} + \underbrace{\sum \frac{q_\alpha}{m_\alpha}(\nabla p_\alpha + \nabla \cdot \Pi_\alpha)}_{closures},$$

Modified Rutherford Equation for NTMs

Pressure/curvature

Neoclassical current

$$
\begin{aligned}
0.41\frac{\partial W}{\partial t} \quad = \quad & D_R^{neo}\left[\frac{\Delta_c'}{4} - \frac{19.5}{W}\frac{\epsilon L_s^2}{B_0^2}\frac{\partial p(0)}{\partial \psi} + 0.58\frac{\sqrt{\epsilon}\beta_\theta \frac{L_q}{L_p}}{W}\frac{W^2}{W^2+W_\chi^2}\right. \\
& \left. + \frac{L_s^2}{k_\theta^2 v_A^2}\left(2.3\frac{(\omega-\omega_E)(\omega-\omega_E-\omega_*)}{W^3} + 0.24\frac{{\omega_E'}^2}{W}\right) - 0.77\frac{L_s}{k_\theta v_A}\frac{\bar{v}_{||0}}{v_A}\frac{\omega_E'}{W}\right]
\end{aligned}
$$

differential flow

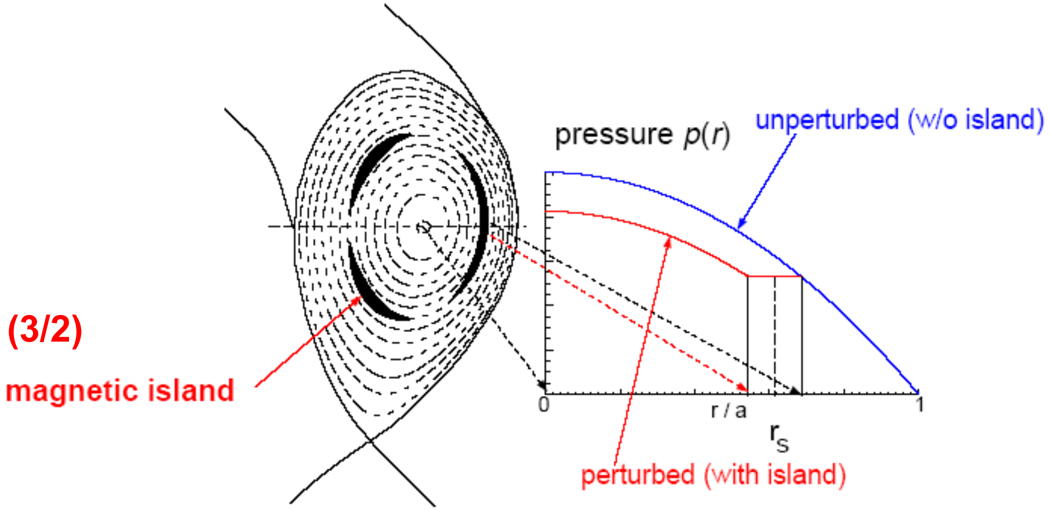
flow shear

polarization current

Effects of NTMs

- **Can degrade confinement** – fast temperature flattening across island due to high parallel thermal conductivity

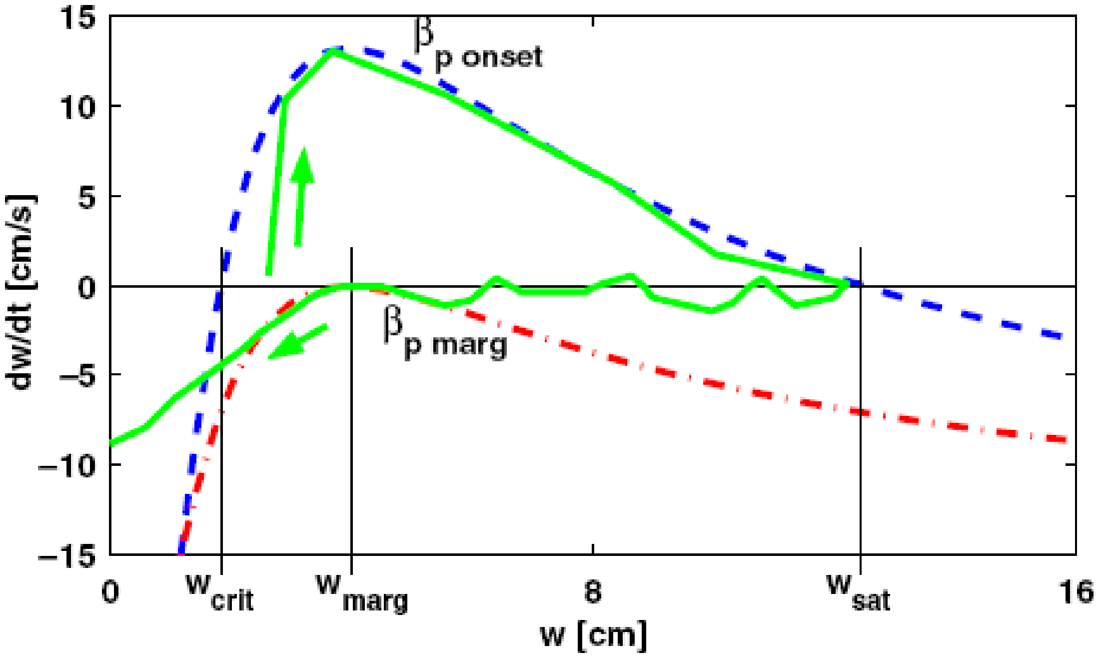
$$\frac{\Delta\tau_E}{\tau_E} = 4\frac{w\rho_s^3}{a^4}$$



- Can cause **disruption** if island size becomes comparable to distance between mode rational surface and plasma edge (depends on beta_poloidal)

O. Sauter, Erice 2005

Time evolution of an NTM growth rate



O. Sauter, Erice 2005

Brief Survey of Experimental Observations on NTMs

Experimental observation of NTMs

- Earliest observations were on TFTR - in supershot discharges

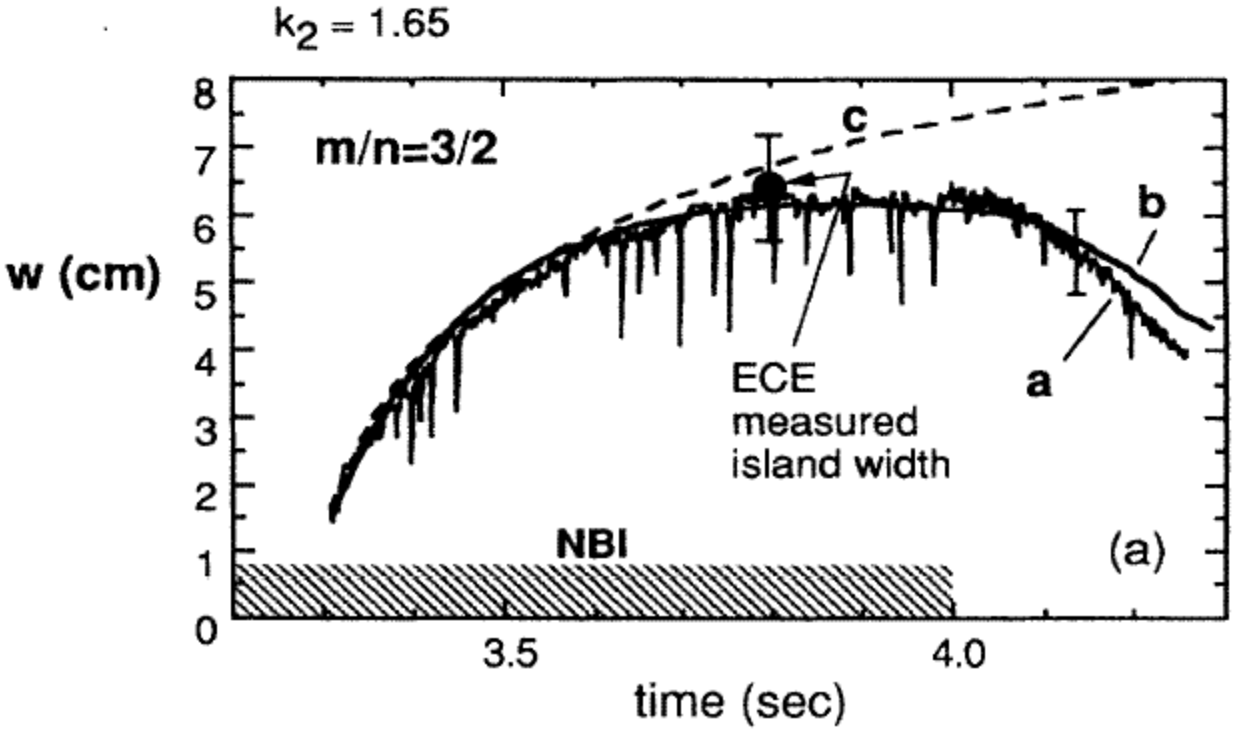
- **Mainly (3/2) or (4/3) modes with f<50khz**

- Degradation of plasma performance with growth of NTM

- Characteristics agreed quite well with Rutherford model estimates

(Z. Chang et al, PRL **74** (1995) 4663)

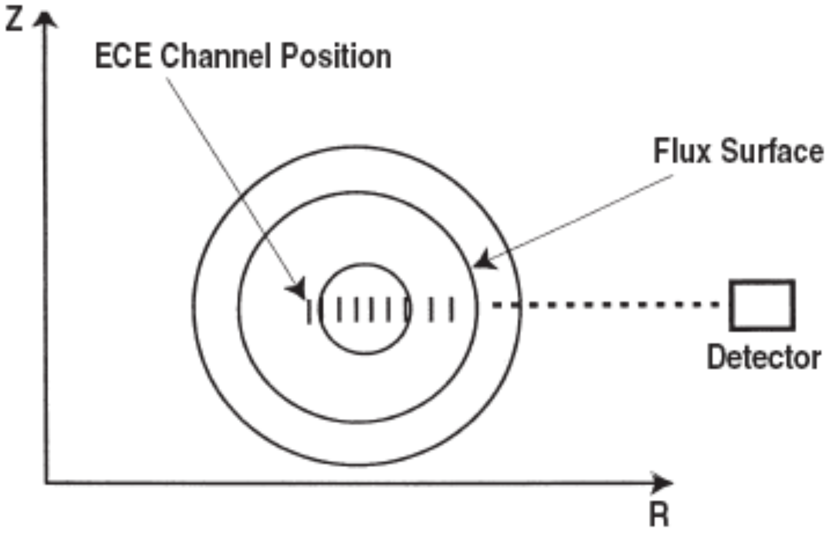
TFTR



Comparison of “measured” island widths with Rutherford model estimates.

Island Structure Can be Measured by Electron Cyclotron Emission of T_e Fluctuation Radial Profile

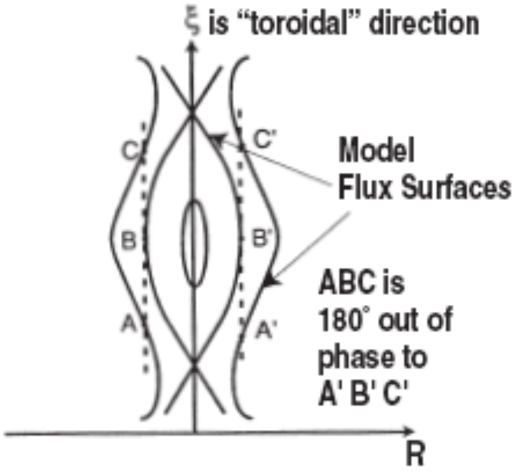
- Magnetic surface distortion
  - ★ leads to T_e fluctuation

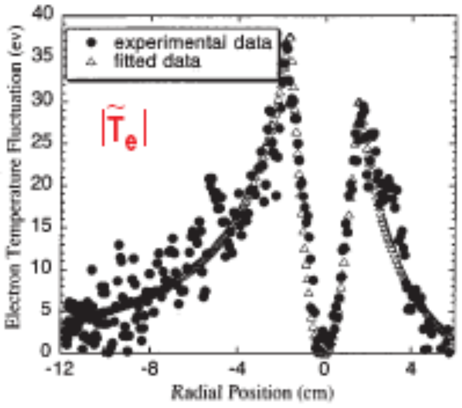


(Y. Nagayama et al., 1990)

w also measured by magn. Probes:

$$w = 4\sqrt{\frac{q\psi}{q' B_{\theta_s}}} = 4\sqrt{\frac{R_0 q}{B_0 s}} \rho_s^m \delta B_{\theta,mn,edge}.$$

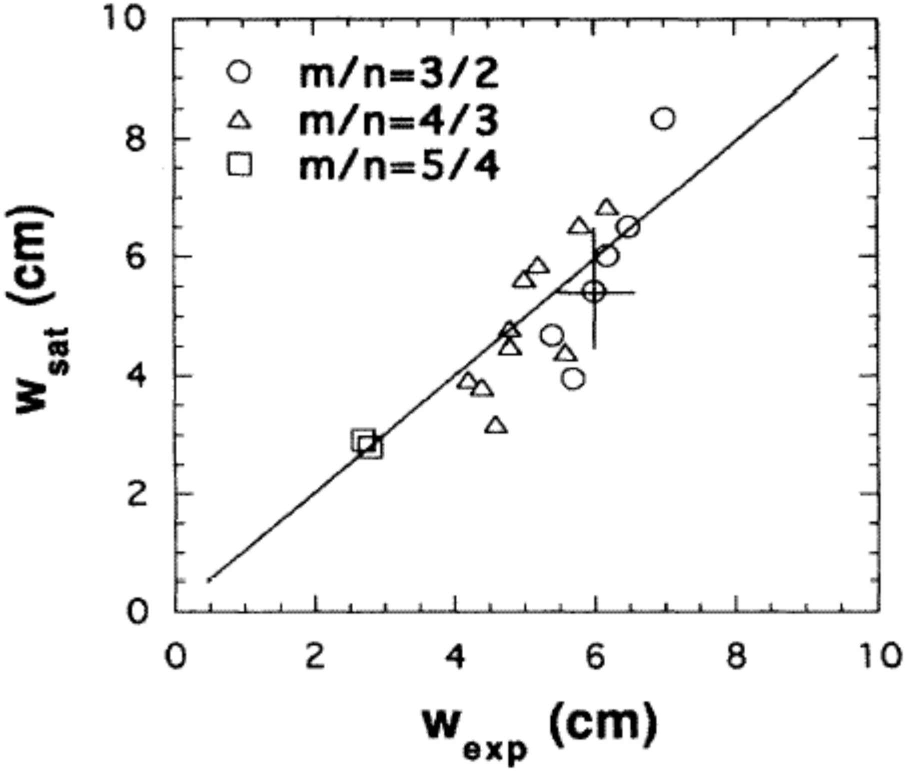




TFTR
#82443
m/n=3/2
$f_{mode} \approx 10$ kHz
(Fitted $\Delta' r_s = -3.7$)

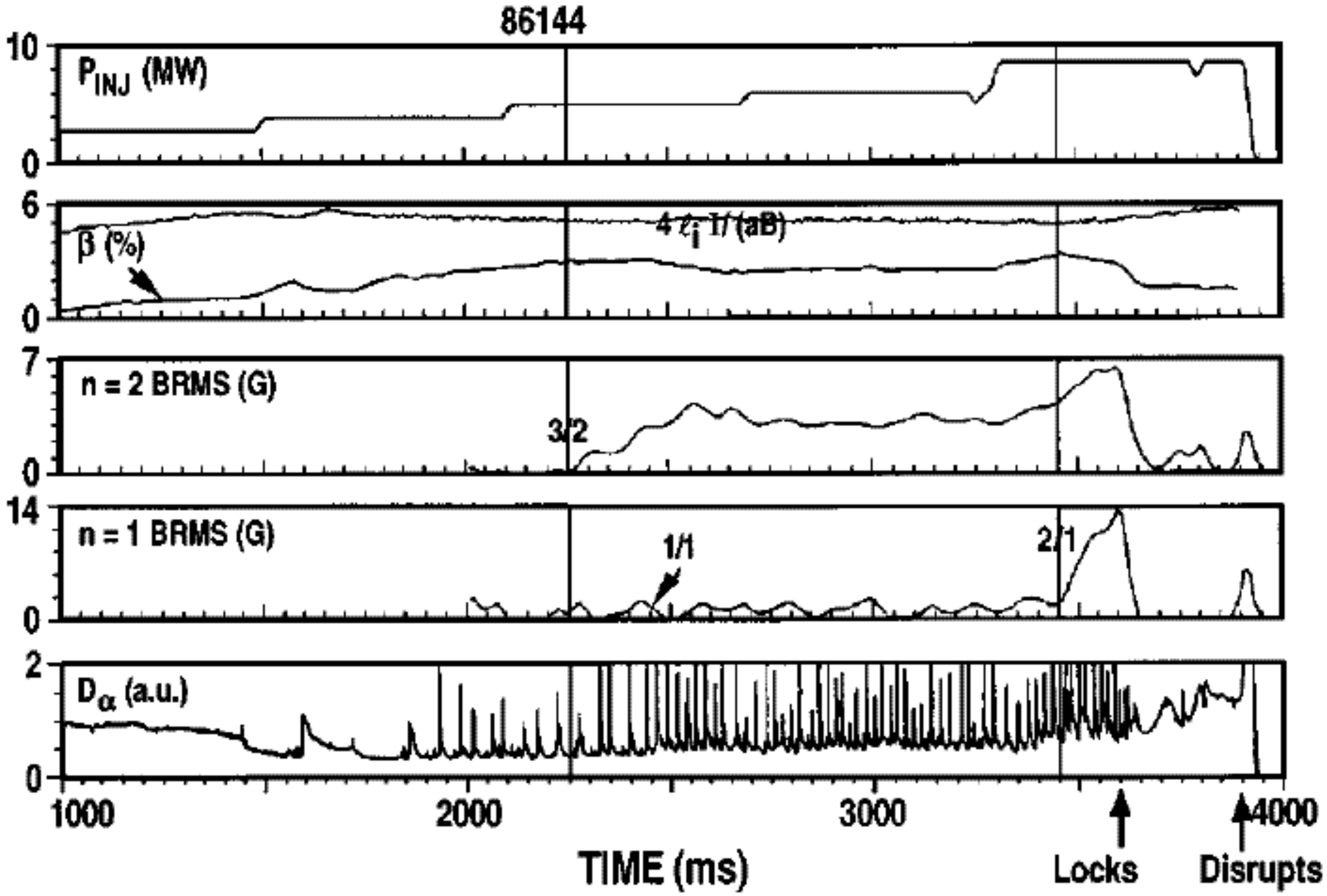
(C. Ren, et., 1998)

TFTR



Theory - experiment comparison of saturated island widths

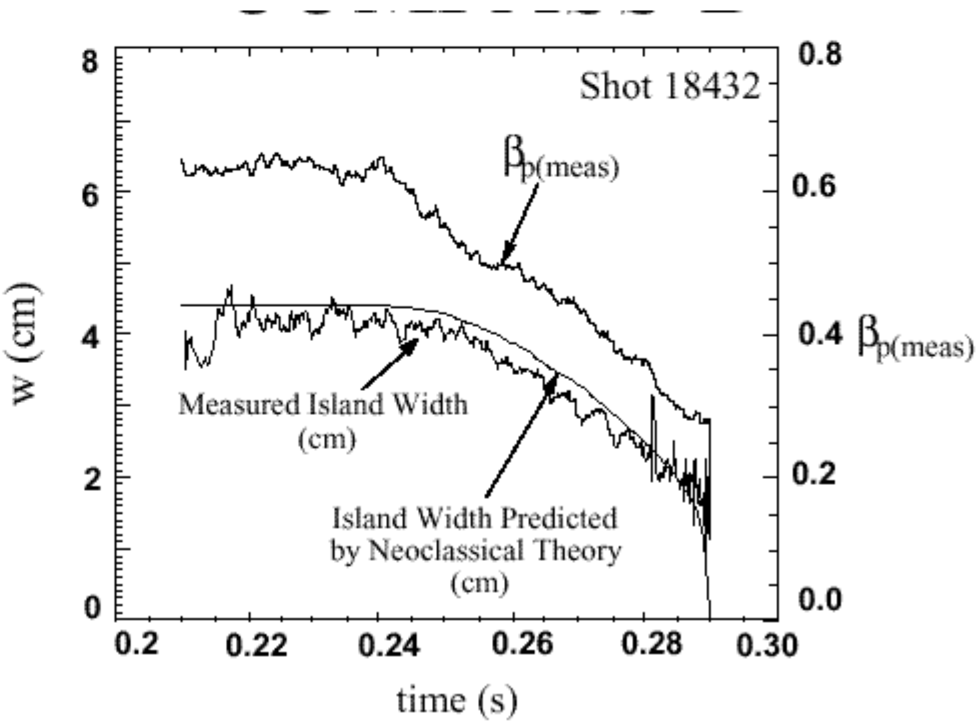
D- III- D observations



A 3/2 mode is excited at t=2250 - saturates beta; at t=3450 a 2/1 mode grows to large amp, locks and disrupts. Ideal beta limit is 3.4

[O. Sauter et al, PoP 4 (1997) 1654]

COMPASS D

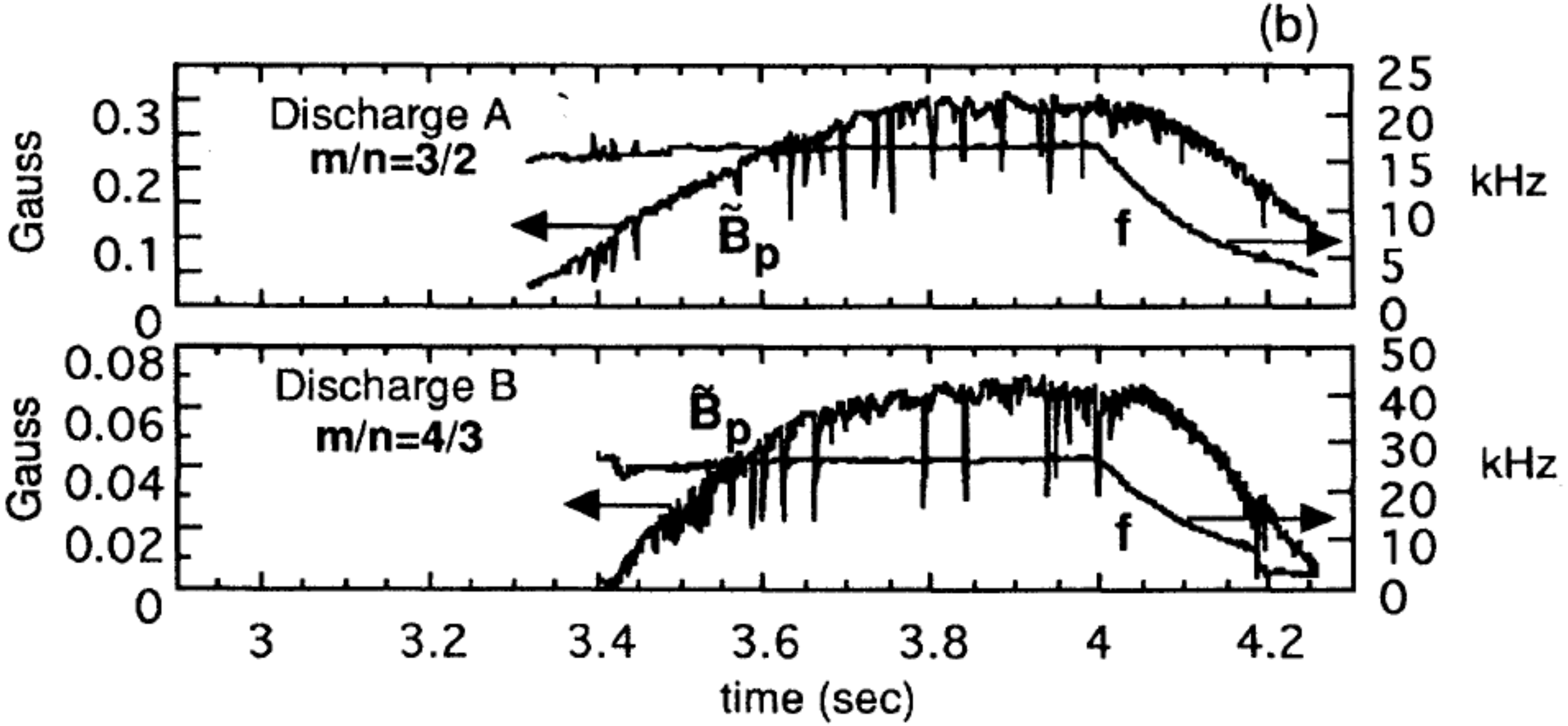


Saturated island width scales like β_p

$$w_{sat} = -a_1 \varepsilon^{1/2} \left(\frac{L_q}{L_p} \right) \frac{\beta_p}{\Delta'}$$

[D.A. Gates et al, Nuclear Fusion **37** (1997) 1593]

TFTR



Single helicity NTMs; f<50 kHz

ASDEX UPGRADE

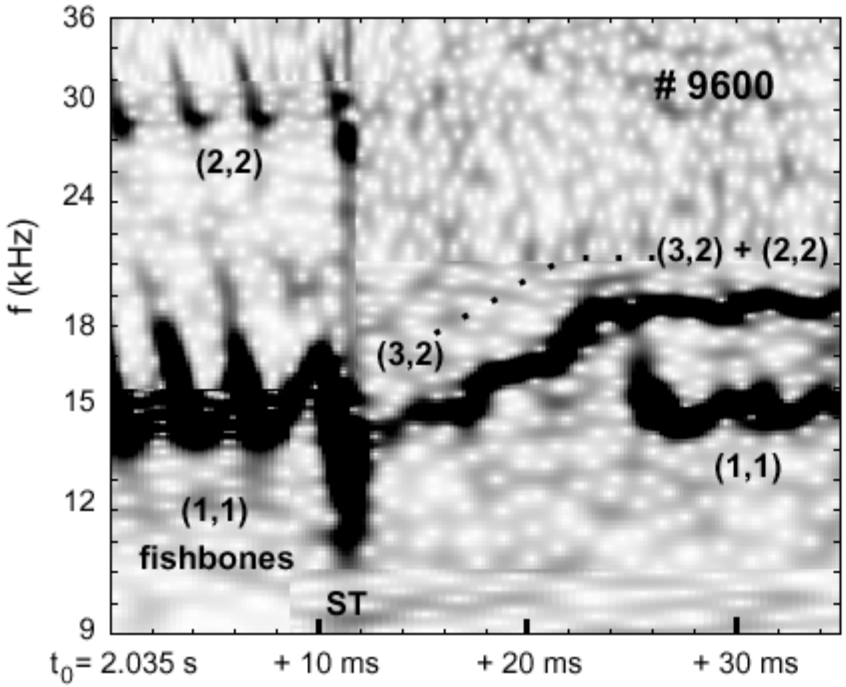


Figure 3. Wavelet plot of an early NTM immediately after a sawtooth crash. The NTM frequency rises during the first 10 ms.

Many experiments have shown a strong correlation between a sawtooth crash and an NTM excitation

ASDEX UPGRADE

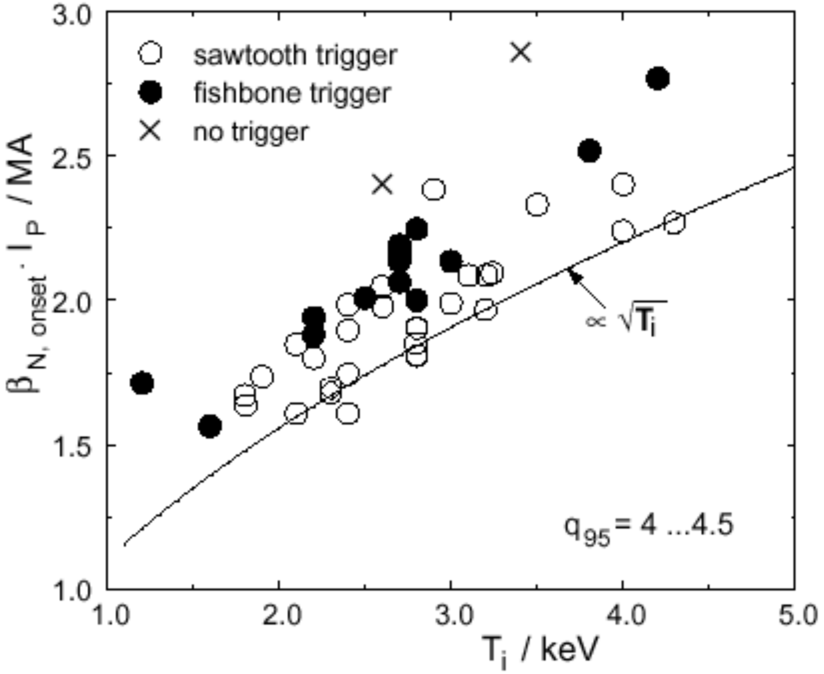


Figure 4. $\beta_{N,onset} \cdot I_p$ vs. the ion temperature at the $(3, 2)$ radial position, T_i. Additionally the scaling, $\beta_{N,onset} \cdot I_p \propto \sqrt{T_i}$, is shown [2].

ASDEX U

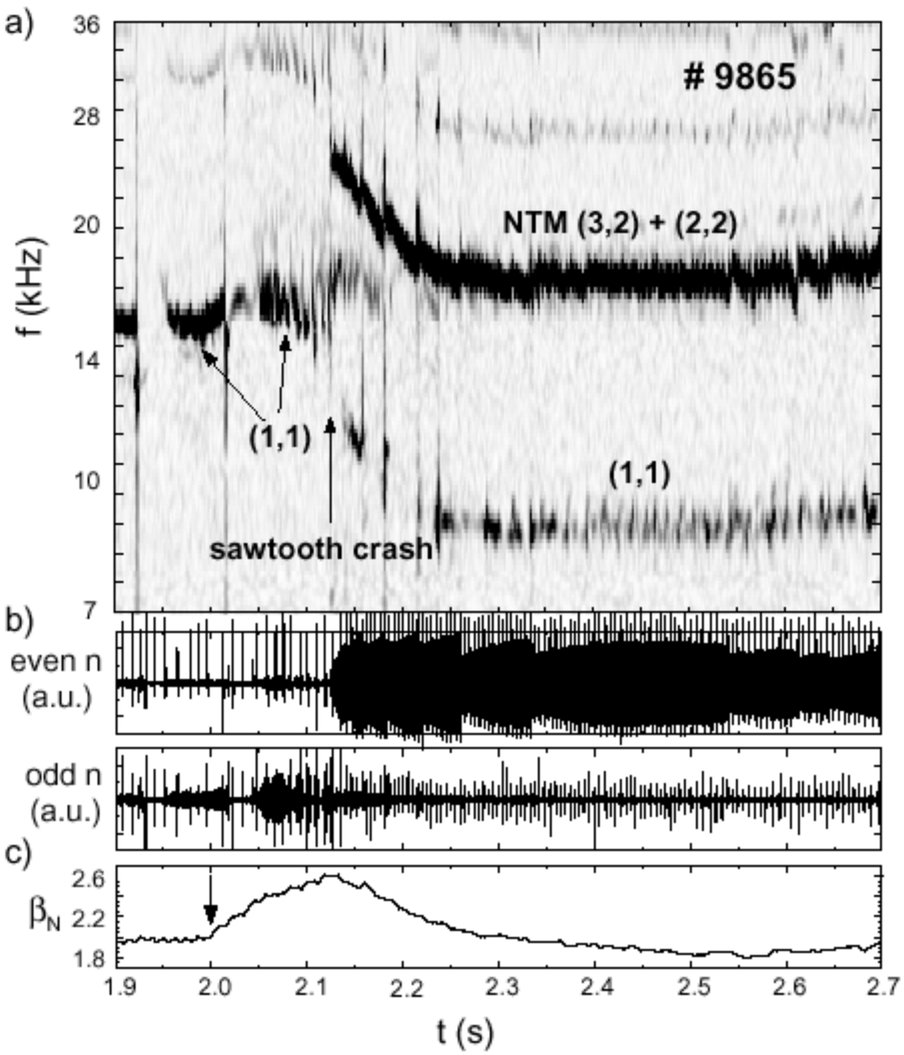
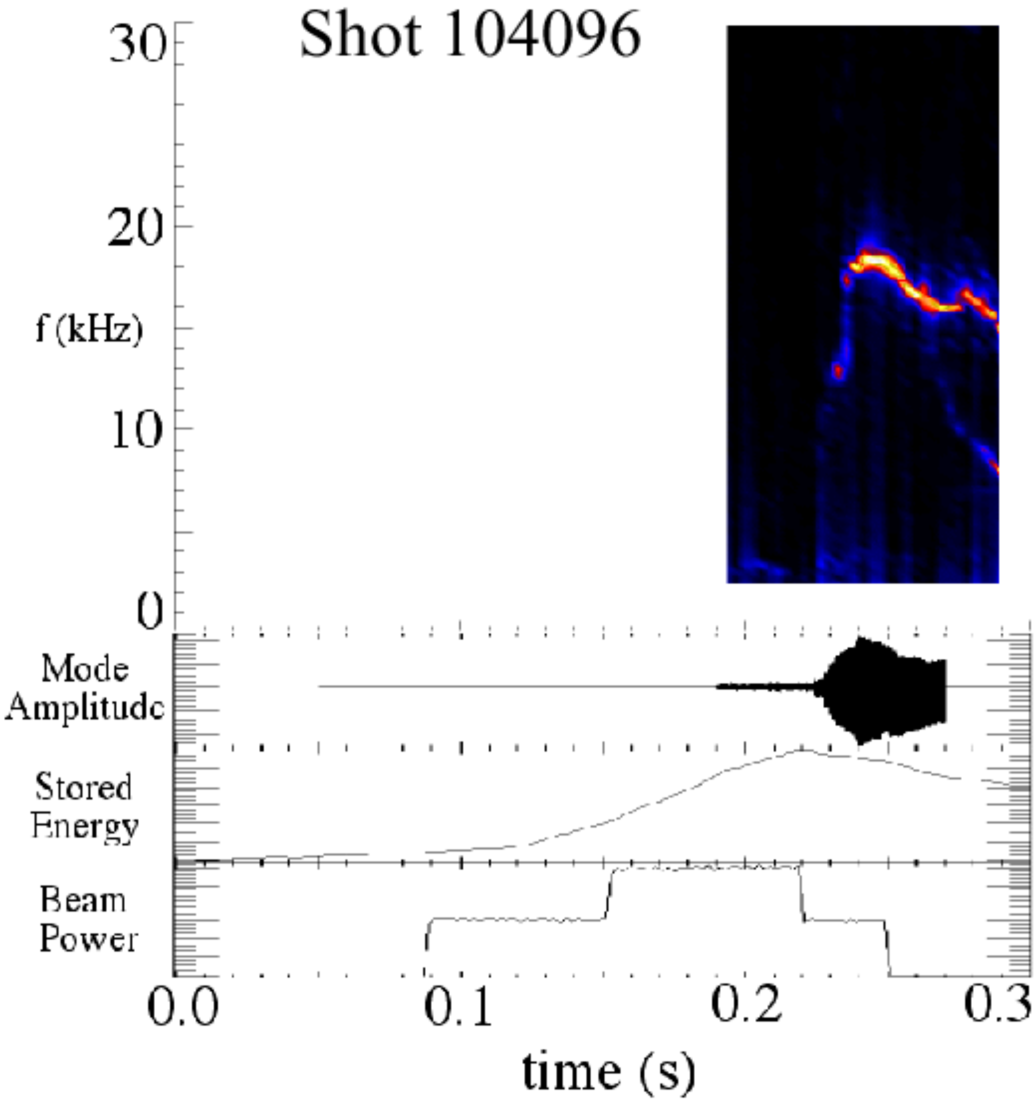


Figure 1. a) Wavelet plot [6] of an NTM. Dark areas represent mode activity. Before the onset of the NTM at 2.126 s fishbone bursts are seen. b) Mirnov signals. The even n signal is dominated by the NTM, the odd n signal by (1,1) modes. c) $\beta_N = \beta_t aB/I$ with $\beta_t = 2\mu_0 p/B_t^2$; the arrow indicates the increase of neutral beam injection power from 5 to 7.5 MW.

NTMs can also be triggered by fishbone activity

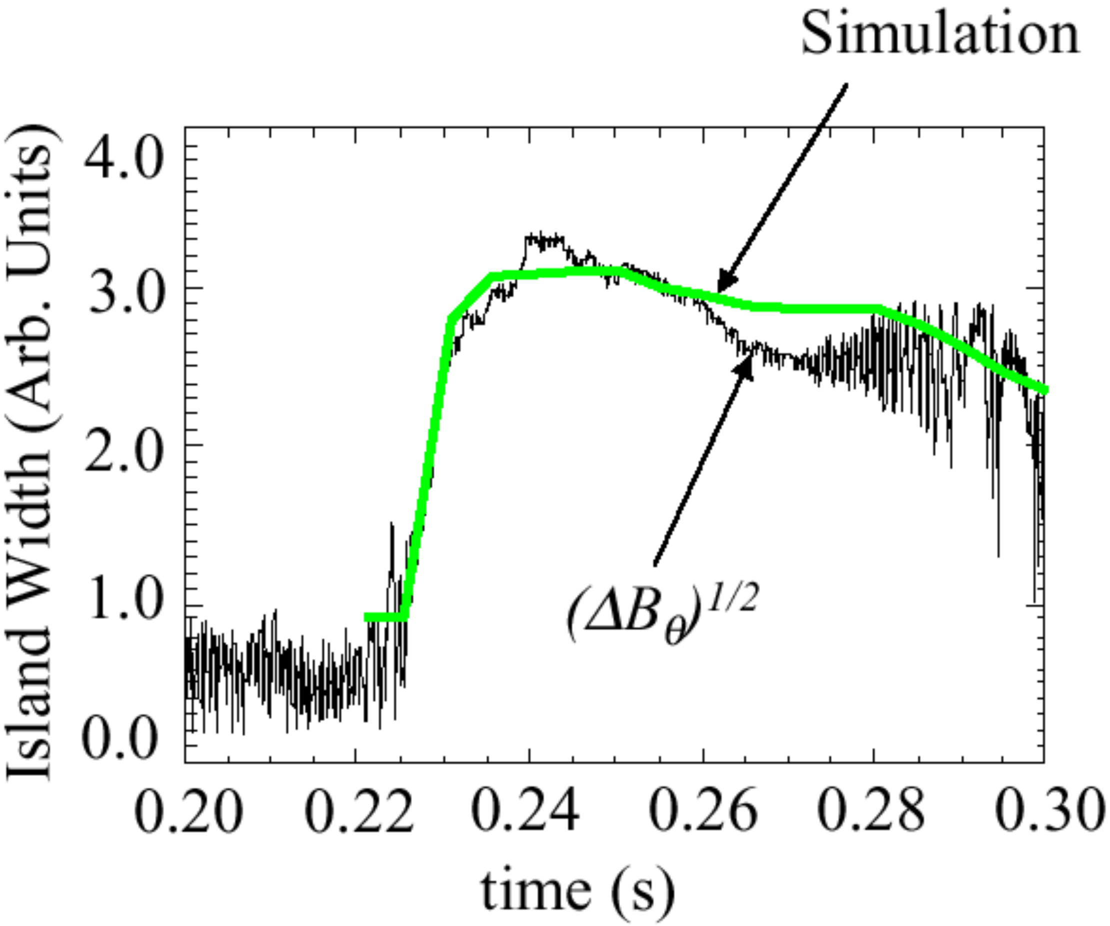
Other triggers: ELMs….

NSTX



- Mode appears at constant poloidal β (β_p ~0.4)
- Slower growth ⇒ resistive mode
- Beam turn off experiment indicates amplitude reduction with stored energy
  - *indicative of bootstrap current driven tearing mode*

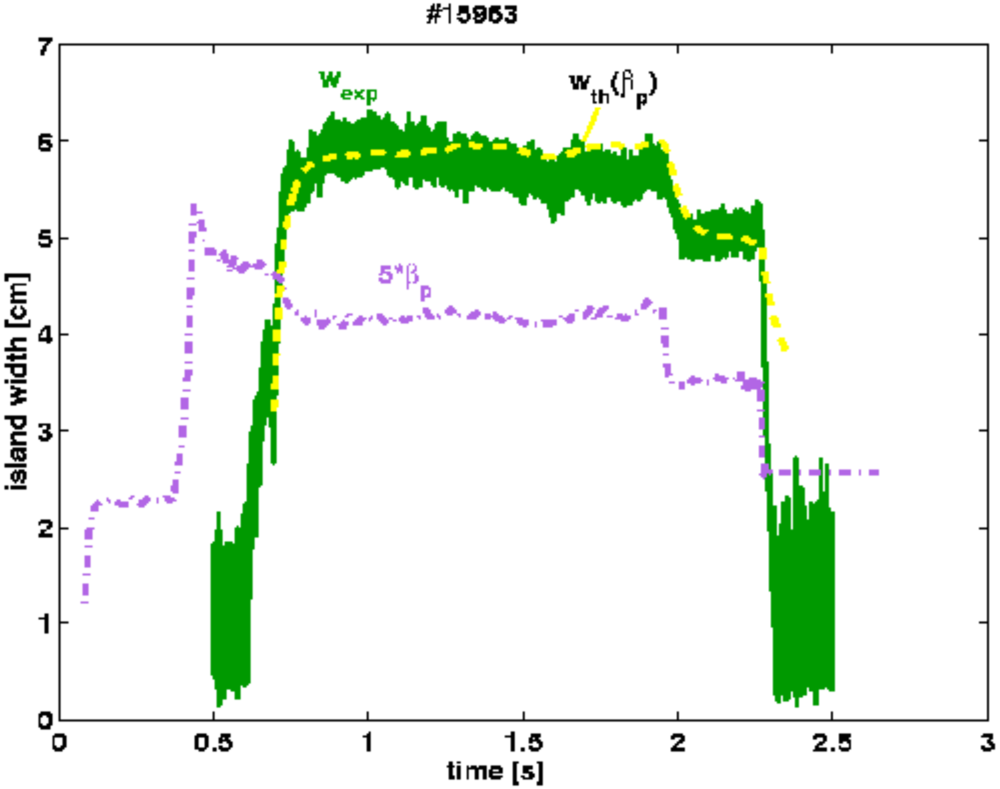
NSTX Results



- Analysis shows reasonable agreement with data
- Interesting amplitude modulation behavior at end of shot
- Use similar values of free parameters as for tokamaks

TCV

Fig. 2 The simulated evolution of a neoclassical 2/1 mode using constant coefficients and the experimental value of β_p *is compared to the experimental island width derived from magnetic measurements.*



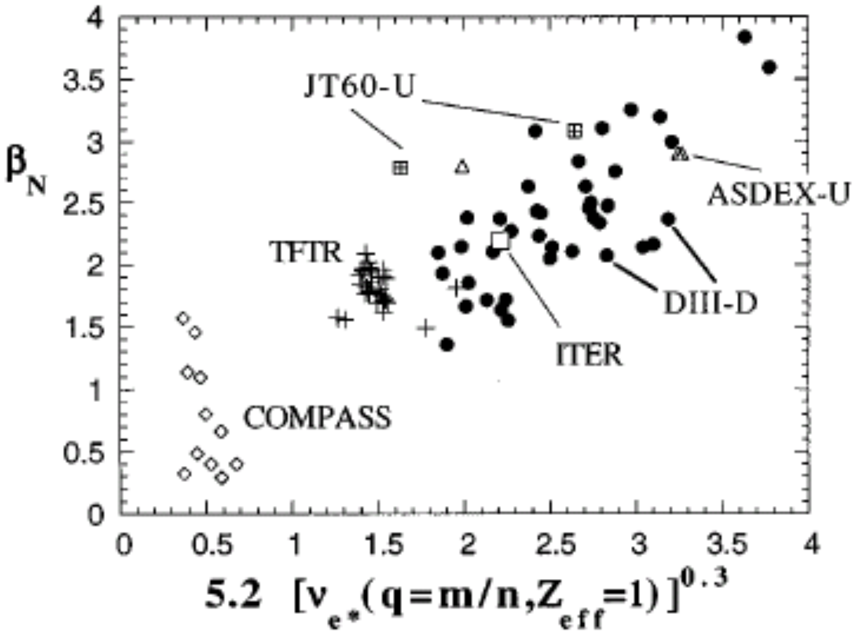
Implications for ITER

- Seed island size ~ 5 to 6 cms

- Saturated island size can be about 60 cms limiting β_N ~ 2.2

- Growth time - 30 s to reach 30 cms & about 150 s to reach 60 cms

- Based on modeling and extrapolation from experiments simulating the ITER parametric regime



How to eliminate or control NTMs?

- Directly control NTMs through appropriate feedback control schemes
    - **ECCD** scheme most successful
- Get to the trigger : prevent sawtooth crash, prevent large ELMs etc
- Other ideas: profile control, rotation, mode coupling etc

How to Stabilize an NTM?

- Principal Idea: **Restore the suppressed bootstrap current within the island**
  - localized current drive -- ECCD, LHCD, NB(?)
  - localized heating - helical temperature variations modify current profile
  - localized density deposition - also changes pressure

- Ohm’s law with auxiliary current

$$\boxed{J_{\|}(\Psi)=\frac{1}{\eta}\langle E_{\|}\rangle+\frac{1}{\eta B}\langle \mathbf{B}\cdot\nabla\cdot\pi_{\|e}\rangle+\langle J_{\mathrm{aux}}\rangle,}$$

- **Modified Rutherford Equation**

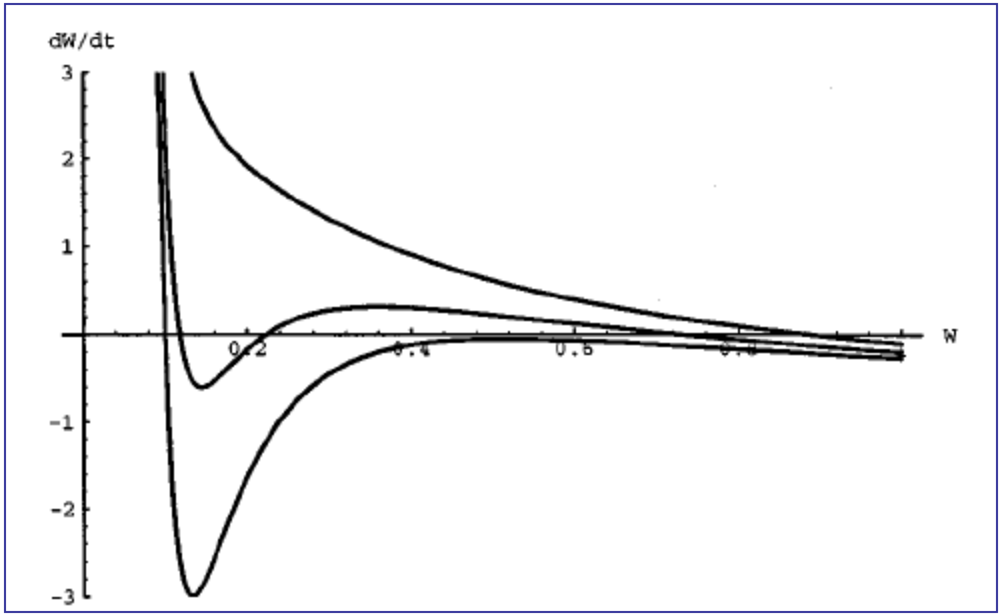
$$\boxed{0.82\frac{dw}{dt}=\frac{1}{\tau_r}\left(\Delta'\rho_s+\frac{D_{nc}}{w}-\frac{D_{\mathrm{aux}}}{w^2}\eta_{\mathrm{aux}}\right),}$$

$$D_{\mathrm{aux}}=\frac{I_{\mathrm{aux}}\mu_0 R}{s\psi_s'\rho_s}\frac{16}{\pi},$$

η_{aux} is an efficiency factor

New “phase diagram”

- Stable and unstable fixed points corresponding to saturated island sizes



$$\eta_{\mathrm{aux}} D_{\mathrm{aux}} > \frac{1}{4}\,\frac{(D_{nc})^2}{(-\Delta'\rho_s)},$$

Condition for complete stabilization

Local Heating Effects

$$\delta J_\parallel = \frac{3}{2}\frac{\delta T_e}{T_{eo}} J_{\parallel o},$$

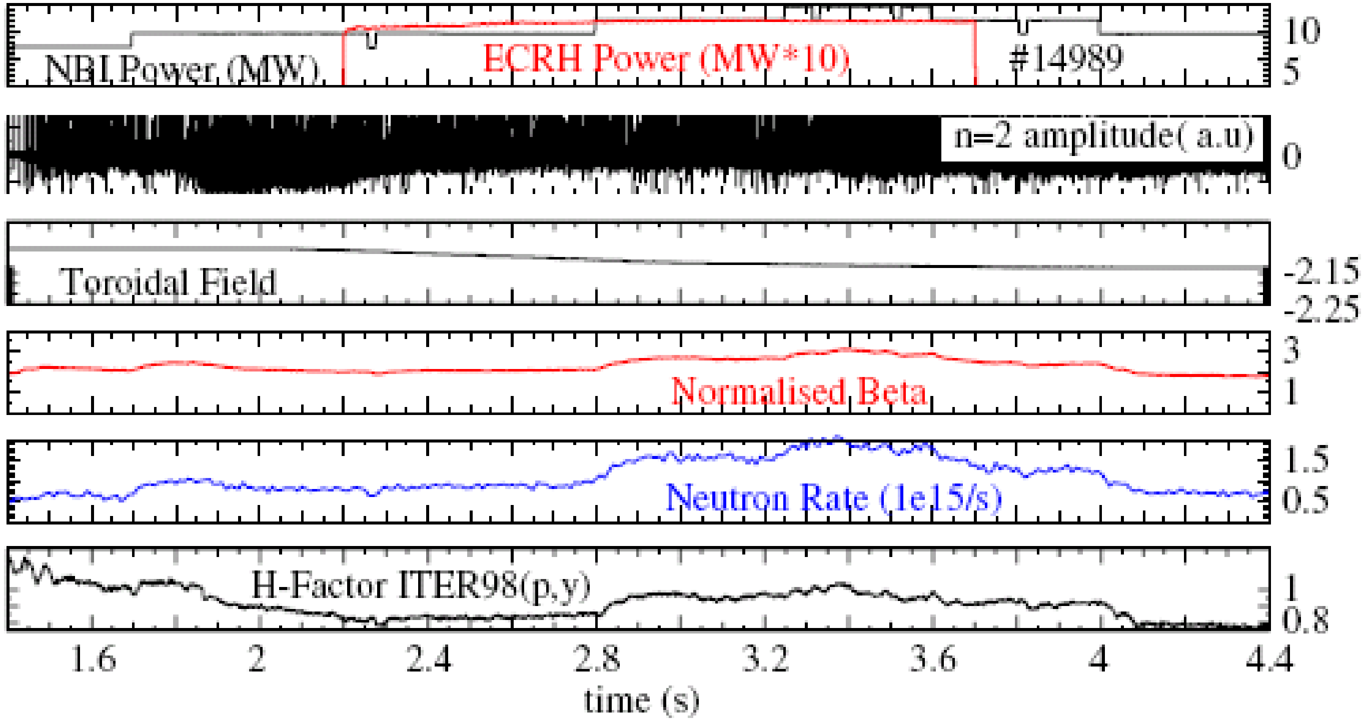
helically resonant temperature variations

$$\boxed{0.82\frac{dw}{dt} = \frac{1}{\tau_r}\left(\Delta'\rho_s + \frac{D_{nc}}{w} - wD_{\text{heat}}\right),}$$

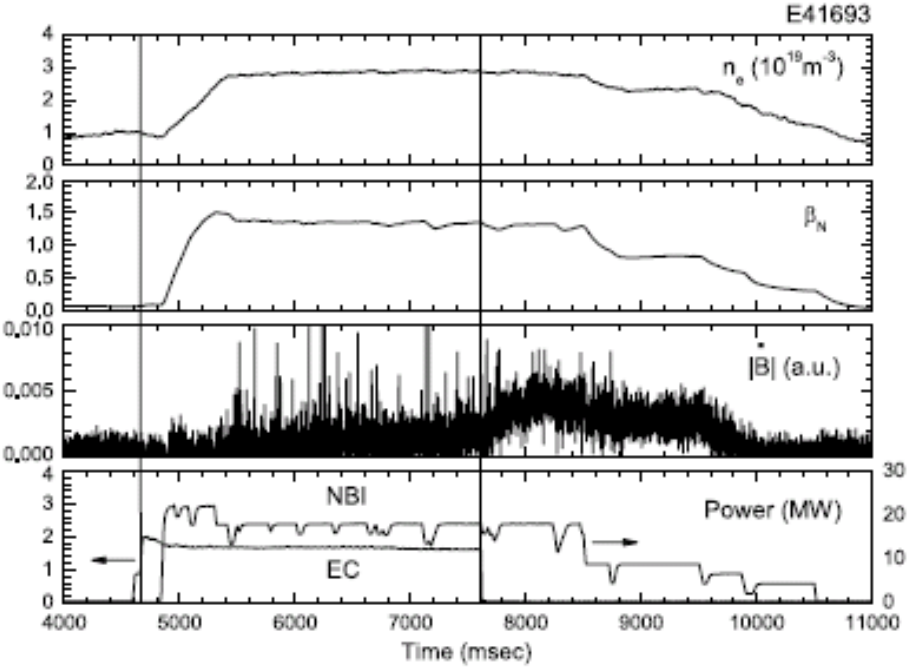
$$D_{\text{heat}} = \frac{16}{5\pi}\frac{q_s}{q'_s}\frac{R\mu_o J_{\parallel o}}{\psi'_s}\frac{S_o\rho_s^2}{nT_e\chi_\perp}$$

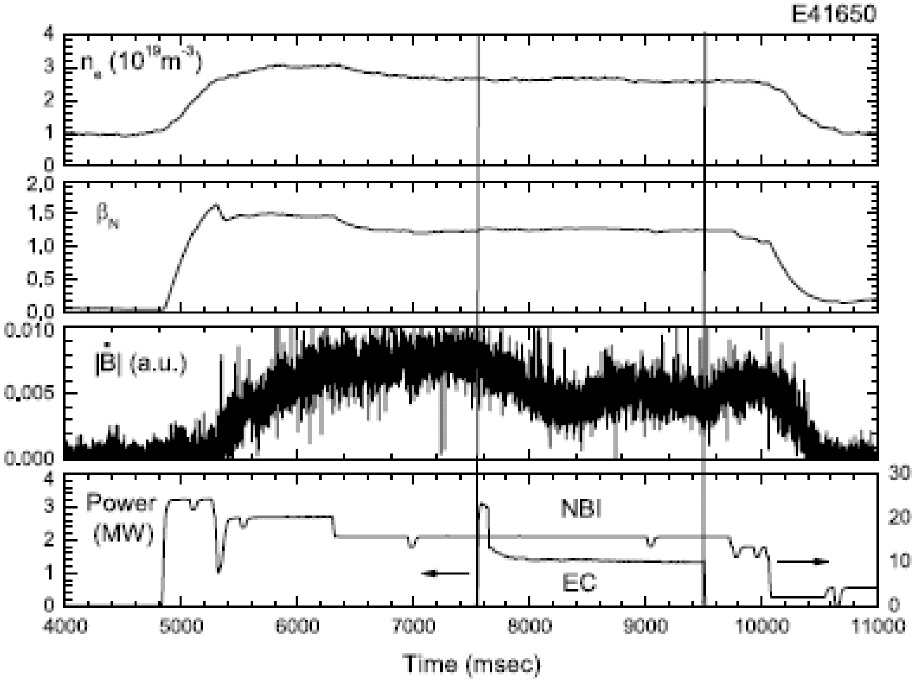
Complete stabilization not possible

$$w_{\text{sat},H} = \frac{D_{nc}}{-\Delta'\rho_s}\frac{2}{1+\sqrt{1+\Upsilon}},$$



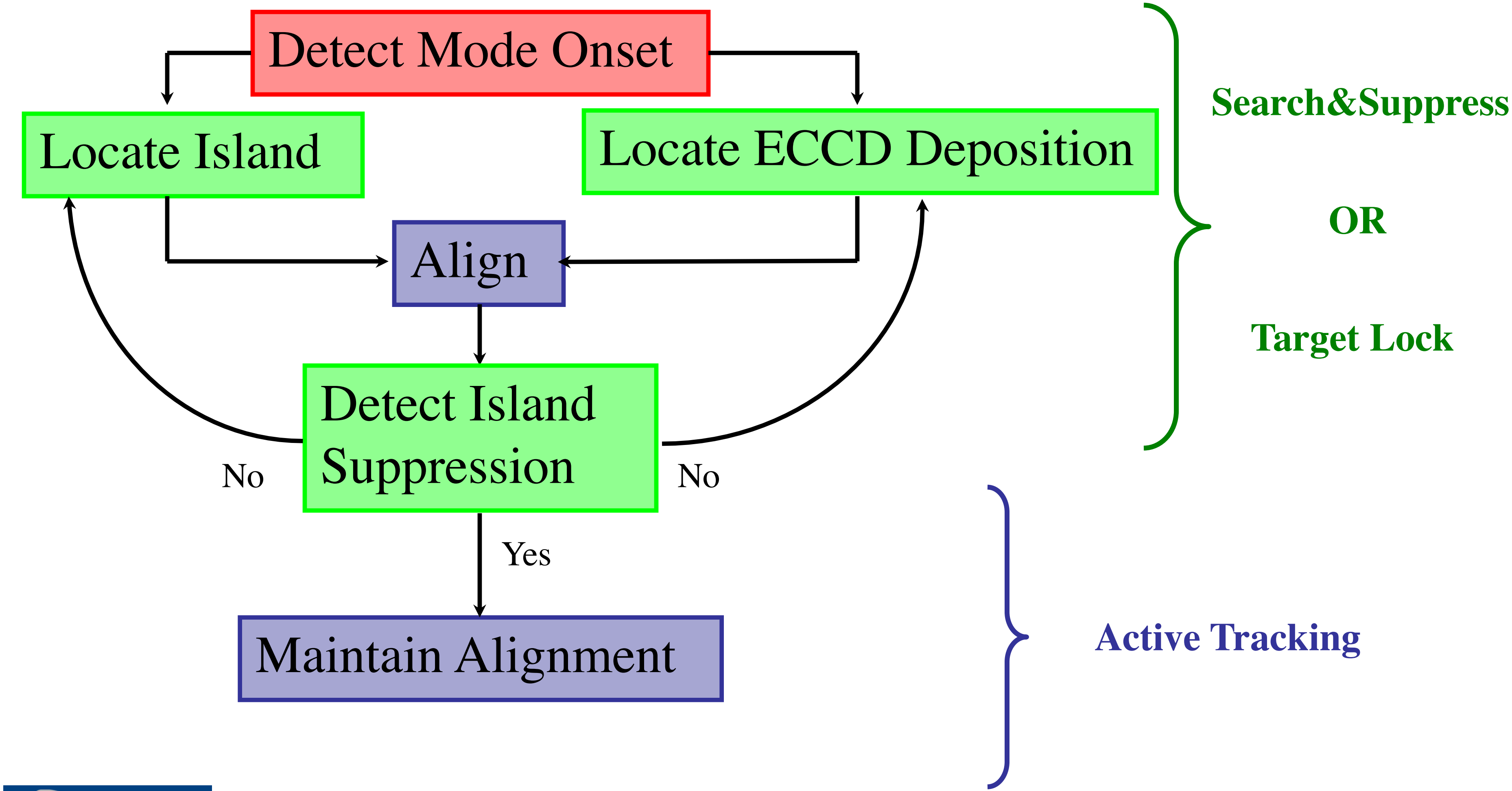
Complete stabilization of a 3/2 NTM in ASDEX-U





Advantage of early application of ECCD in JT60-U

NTM Control Requires Achieving and Sustaining Dynamic Island/ECCD Alignment



Detect Mode Onset

Locate Island

Locate ECCD Deposition

Align

Detect Island Suppression

No

No

Yes

Maintain Alignment

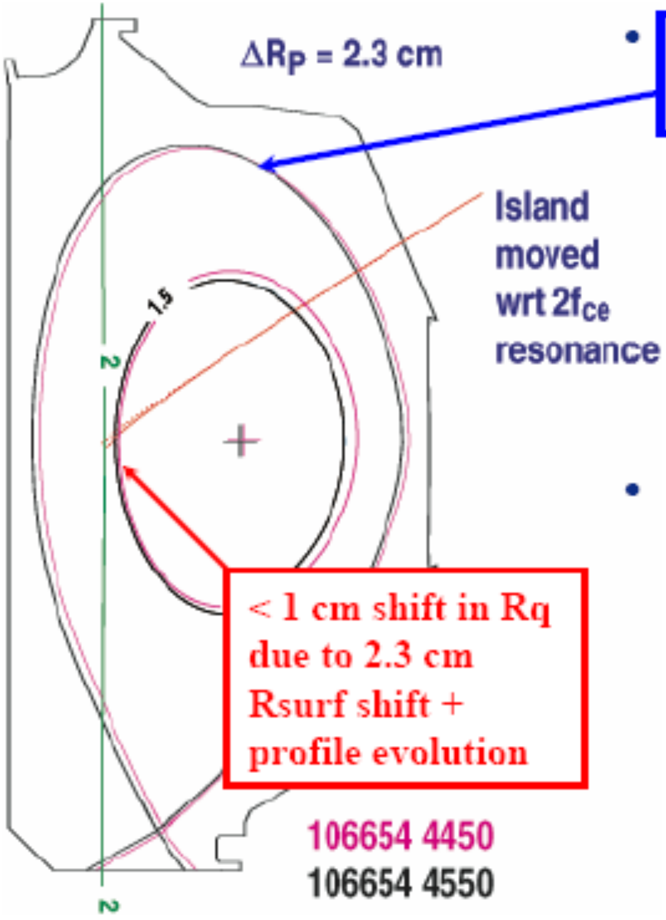
Search&Suppress

OR

Target Lock

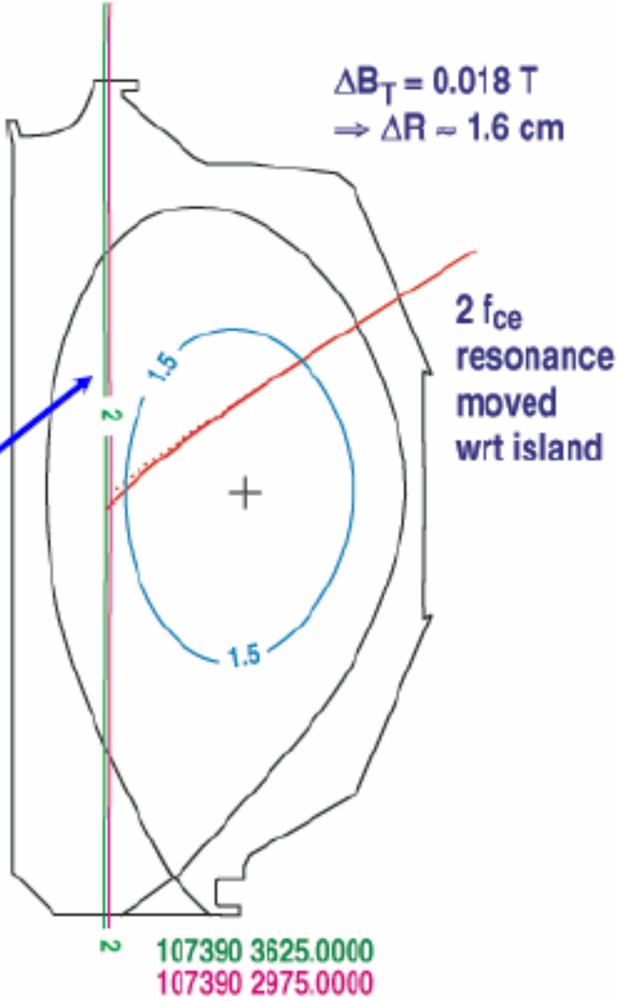
Active Tracking

Actuators: Variation of Plasma Position or Toroidal Field Are Used to Regulate Alignment



< 1 cm shift in Rq due to 2.3 cm Rsurf shift + profile evolution

- Plasma surface major radius can be moved to shift island relative to ECCD
- Toroidal field can be varied to shift ECCD relative to island



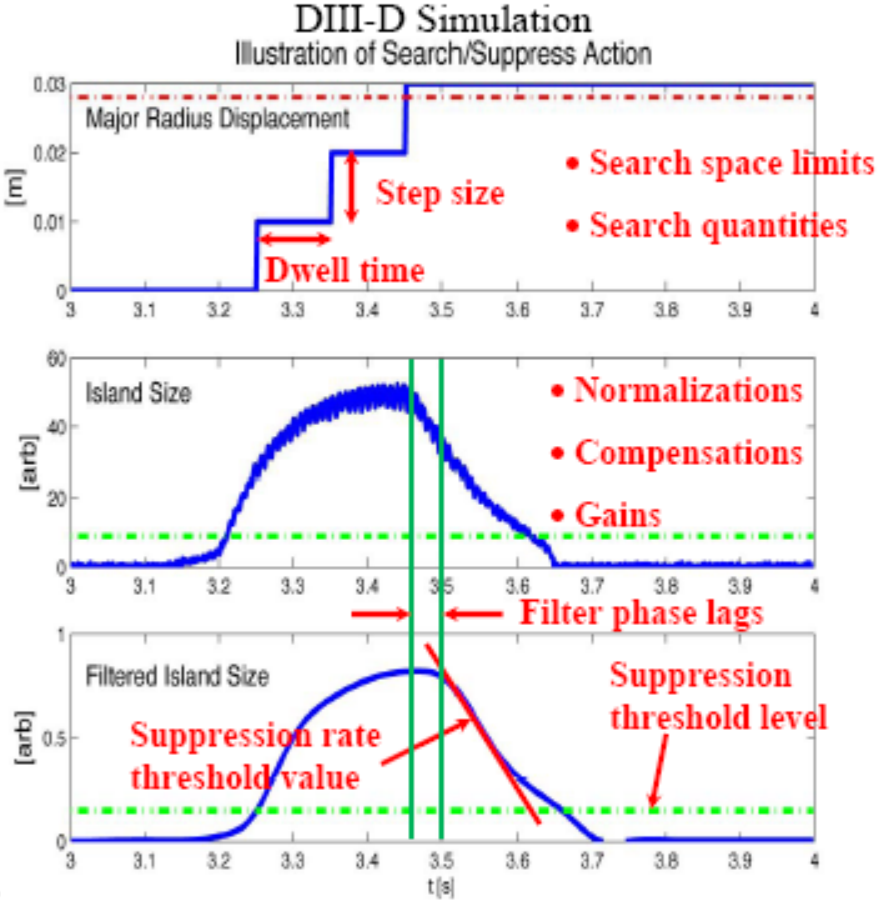
"Search and Suppress" Algorithm Uses Island Response to Detect Island/ECCD Alignment

- **Uncertainty in locations of both island and ECCD comparable to alignment accuracy required (~ 1 cm) ⇒ need systematic search**

- **"Search and Suppress" algorithm:**
  - Vary alignment in steps (e.g. plasma major radius ΔR or toroidal field ΔB_T)
  - Dwell for specified time to measure island response
  - Freeze if island suppressed

- **Adjustable feedback parameters include filters, compensation for plasma motion and rotation**

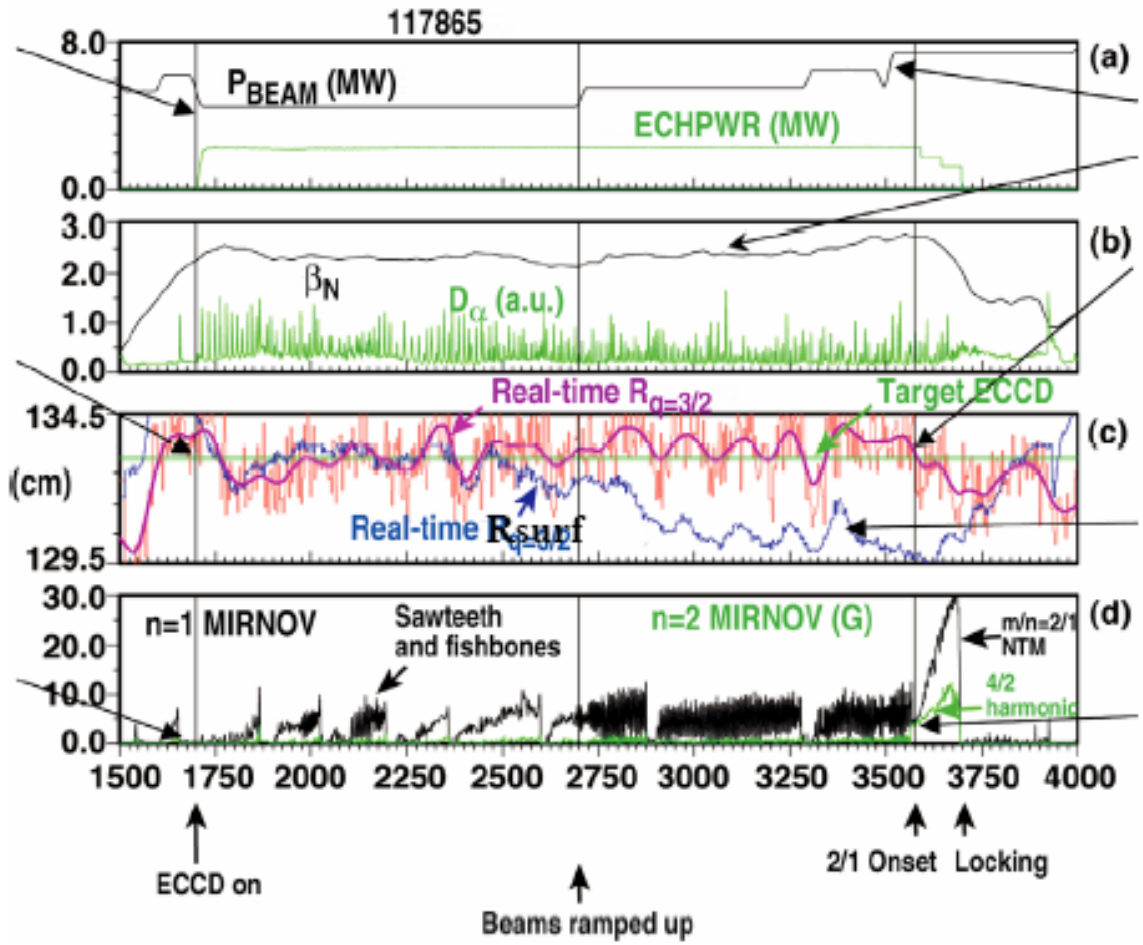
- **Actuator limits prevent plasma-limiter contact**



DIII-D Simulation

Illustration of Search/Suppress Action

Active Tracking of q-Surface Motion Enables Preemptive NTM Suppression



ECCD and control enabled

ECCD initially ~ aligned with q=3/2 surface

No NTM initially

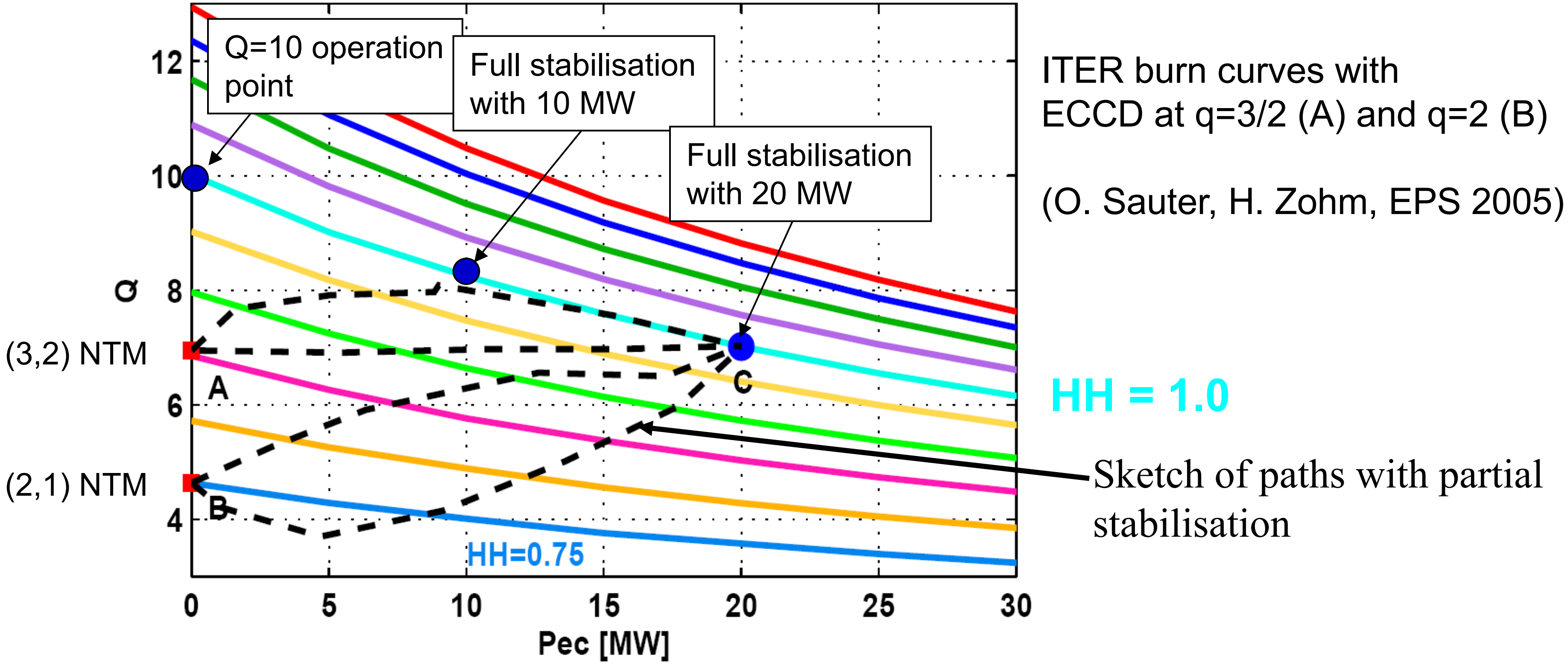
Beam power and β_N are increased…

..alignment is maintained with realtime q=3/2 surface reconstruction..

…by moving plasma major radius rigidly

3/2 NTM suppression sustained

ITER NTMs stabilisation goals



ITER burn curves with ECCD at q=3/2 (A) and q=2 (B)

(O. Sauter, H. Zohm, EPS 2005)

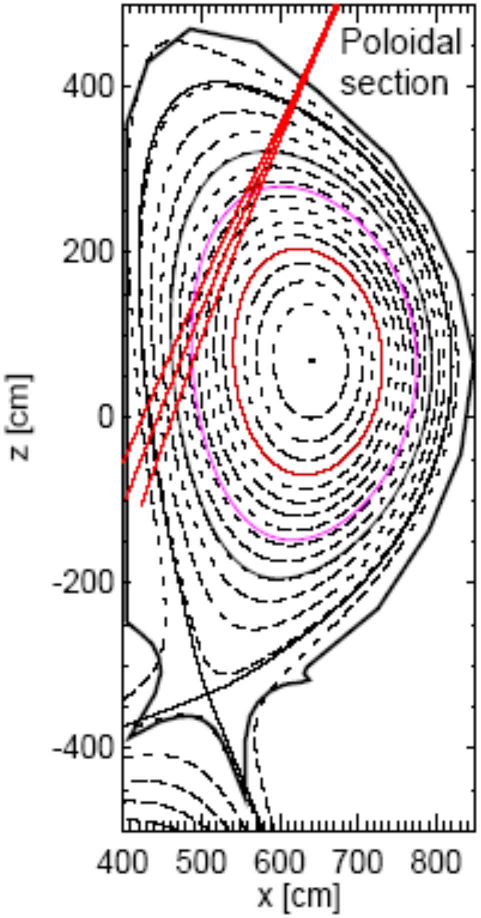
Sketch of paths with partial stabilisation

Impact on Q in case of continuous stabilisation (worst case):

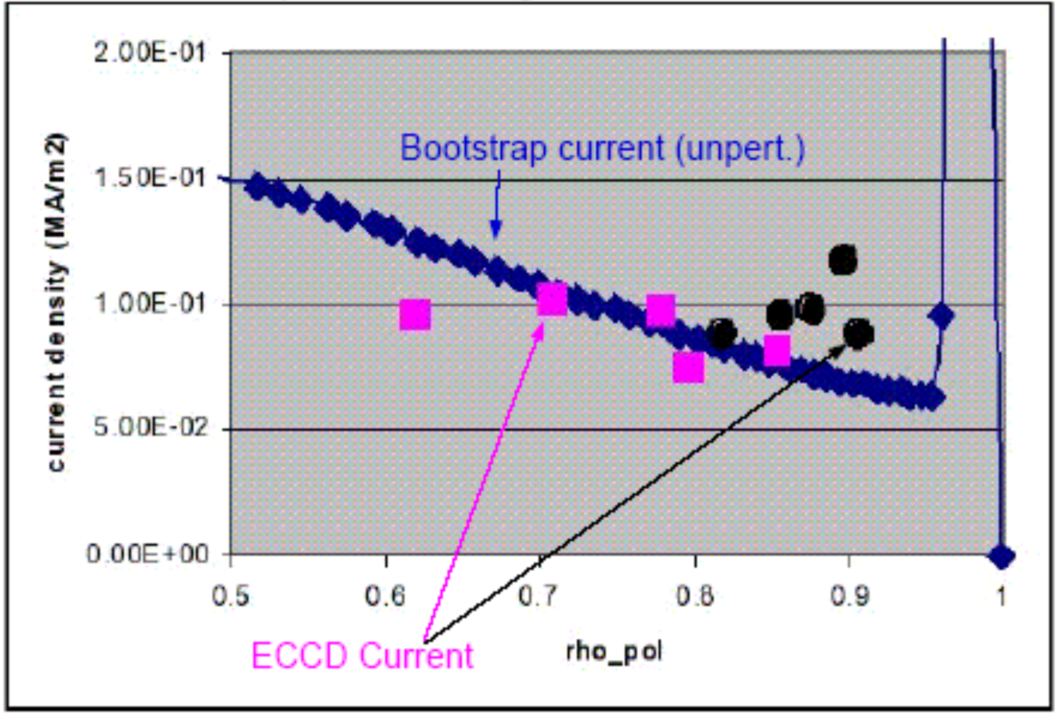
- Q drops from 10 to 5 for a (2,1) NTM and from 10 to 7 for (3,2) NTM
- with 20 MW needed for stabilisation, Q recovers to 7, with 10 MW to Q > 8
- note: if NTMs occur only occasionally, impact of ECCD on Q is small

Active NTM stabilisation in ITER

- Upper ECRH system for active stabilisation of (3,2) and (2,1) islands under development
- Current deposition calculated by means of the TORBEAM code [Poli et al., CPC 1999]

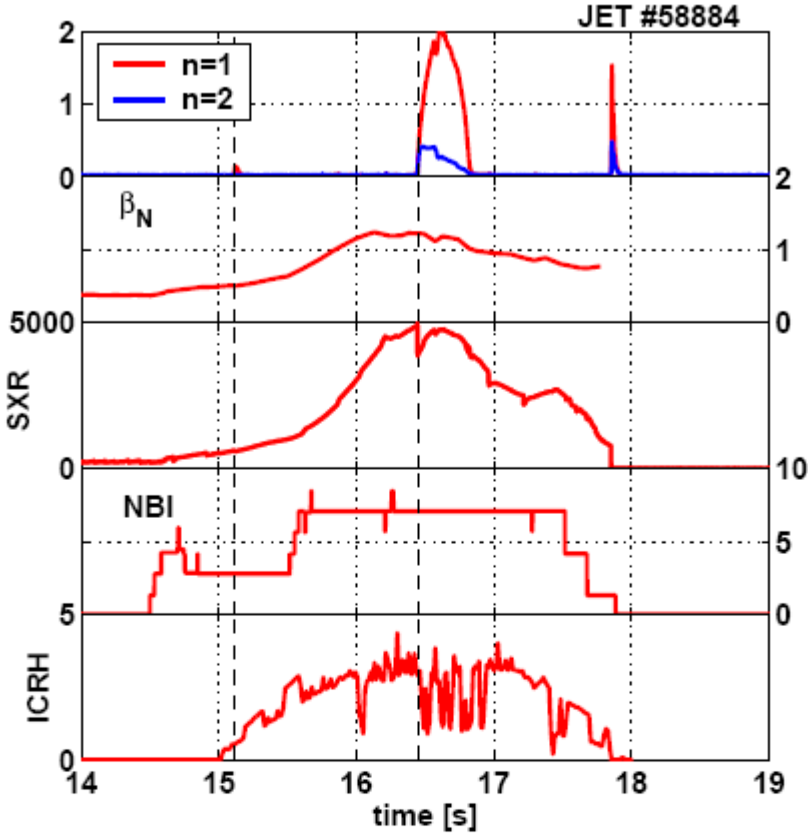


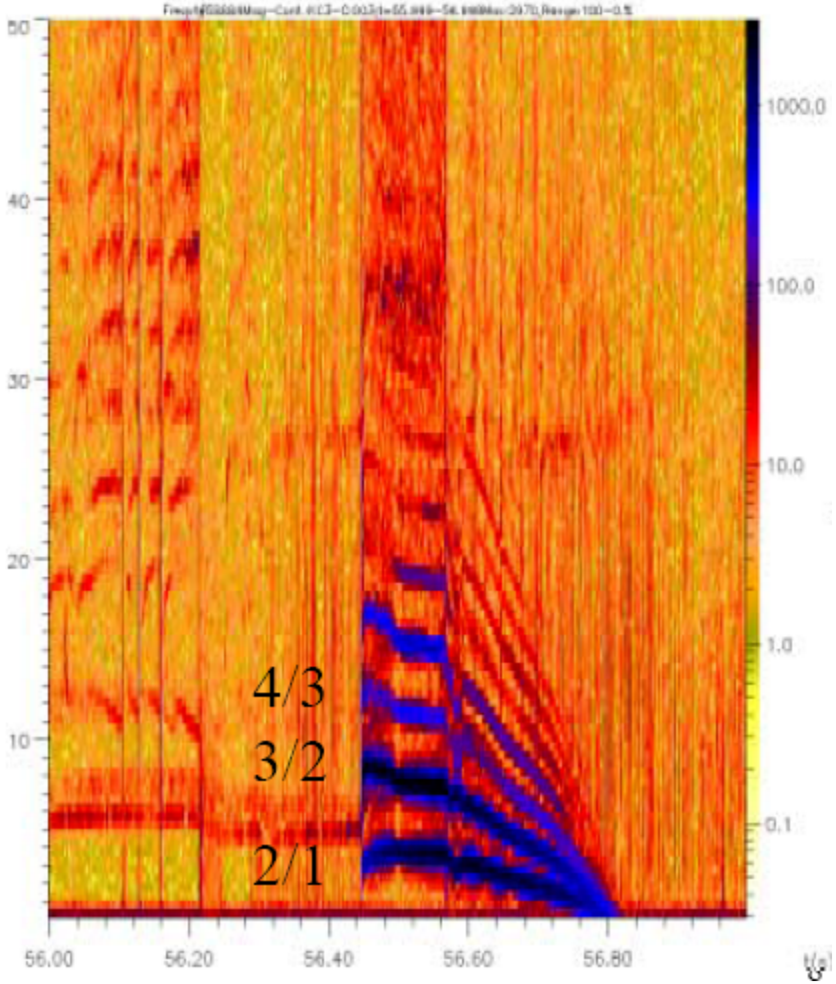
- Driven current smaller than the missing bootstrap current for the present design



[Zohm, Poli et al., EC13 (2004)]

Importance of trigger mechanism (1)





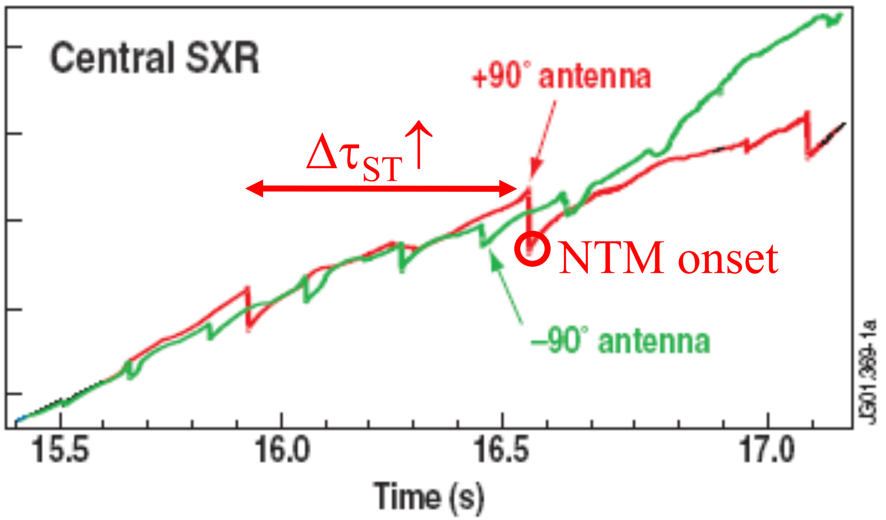
At sawtooth crash, many modes can be triggered

Importance of trigger mechanism (2)

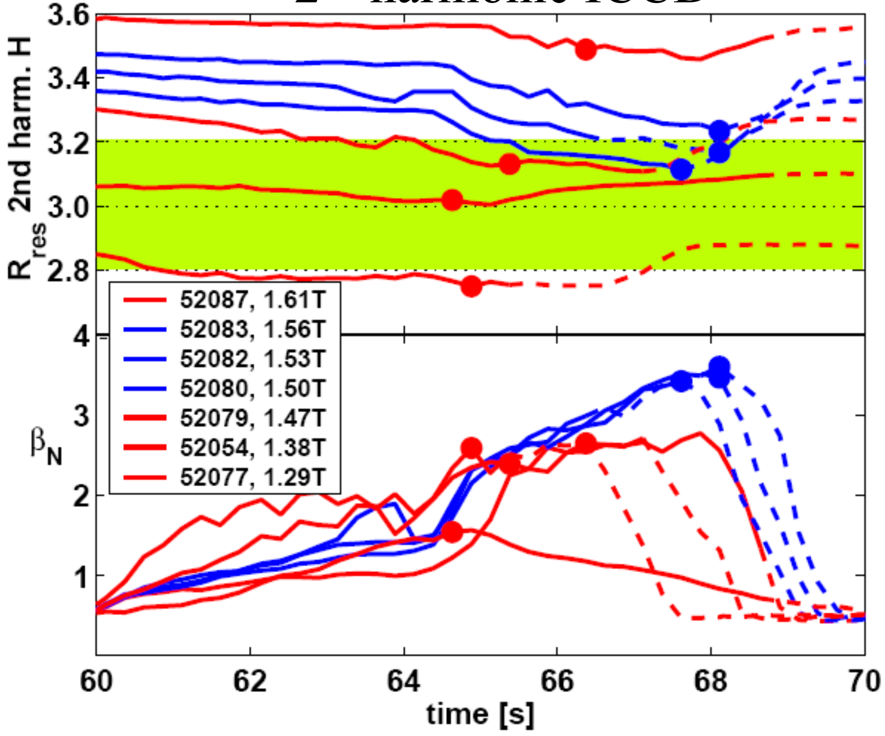
Controlling sawteeth changes significantly β_{onset}

1st harmonic minority ICRH

2.4 T, 2.4 MA, 4.5 MW ICRF, Same NBI



2nd harmonic ICCD



+90: phase: $\beta_{N,onset} \approx 1$

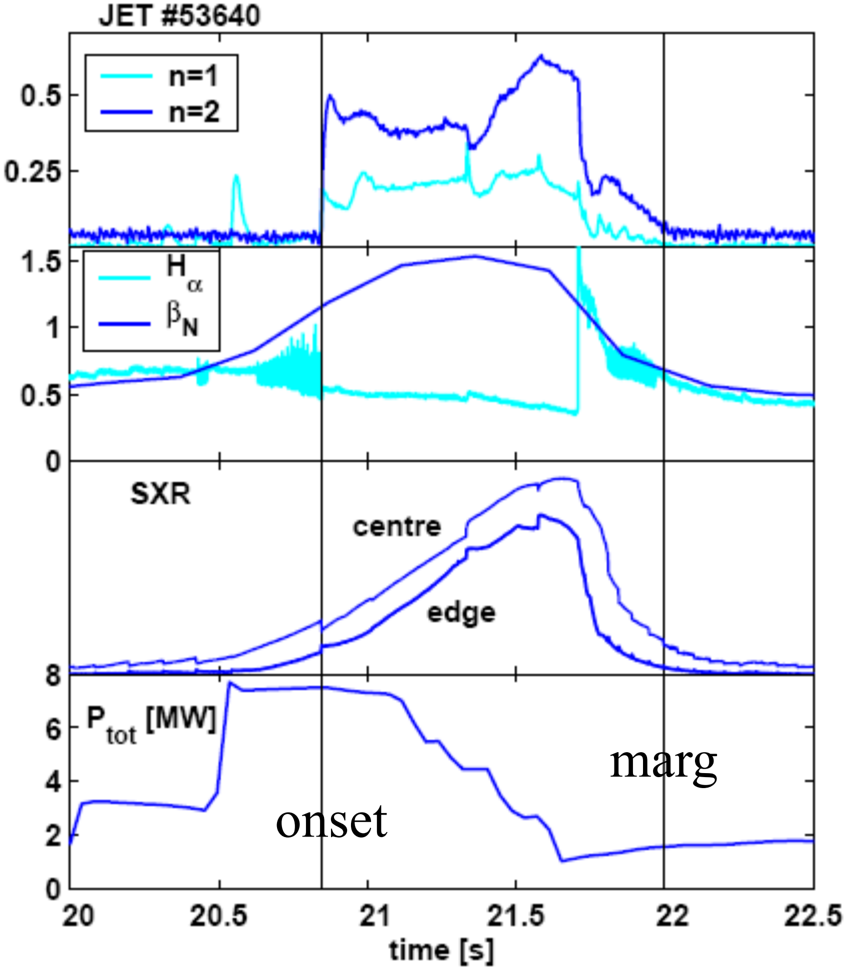
-90: No NTM with β_N up to 2

Sauter et al, PRL 2002

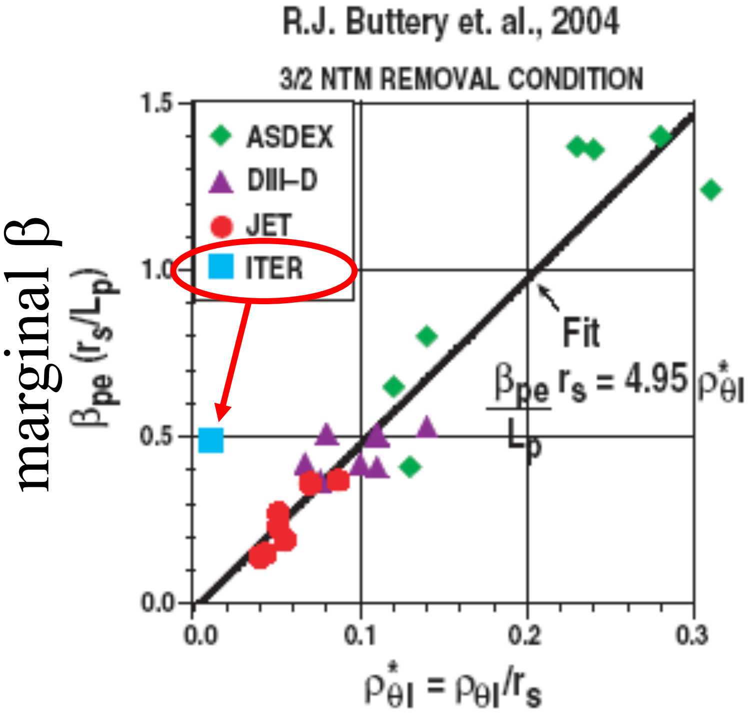
Power ramp-down studies

q_{95}=2.54 case does not disrupt even with 2/1 mode

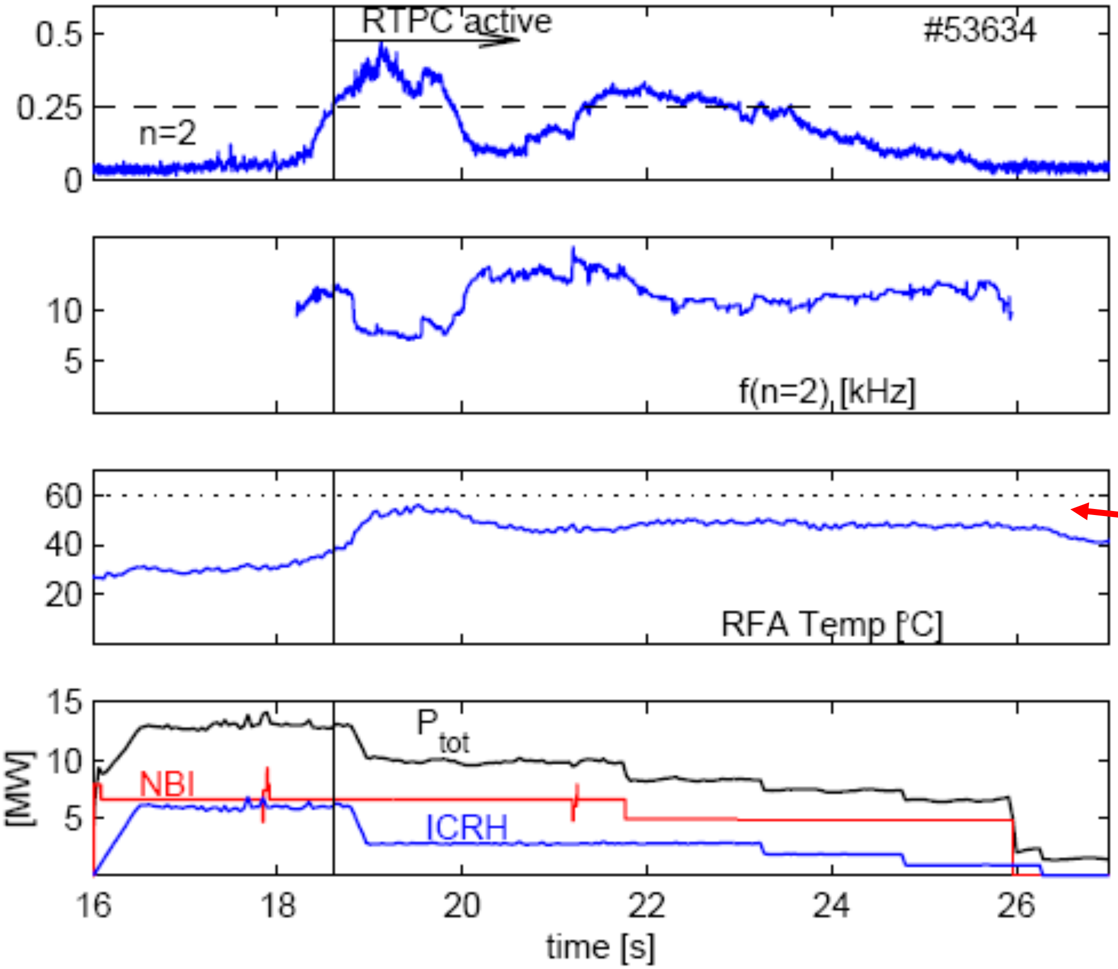
$\beta_{N,marg} <$ L-H threshold



★ $w_{marg} \approx 2\,\varepsilon^{1/2}\,\rho_{\theta i}$
(twice ion banana width)

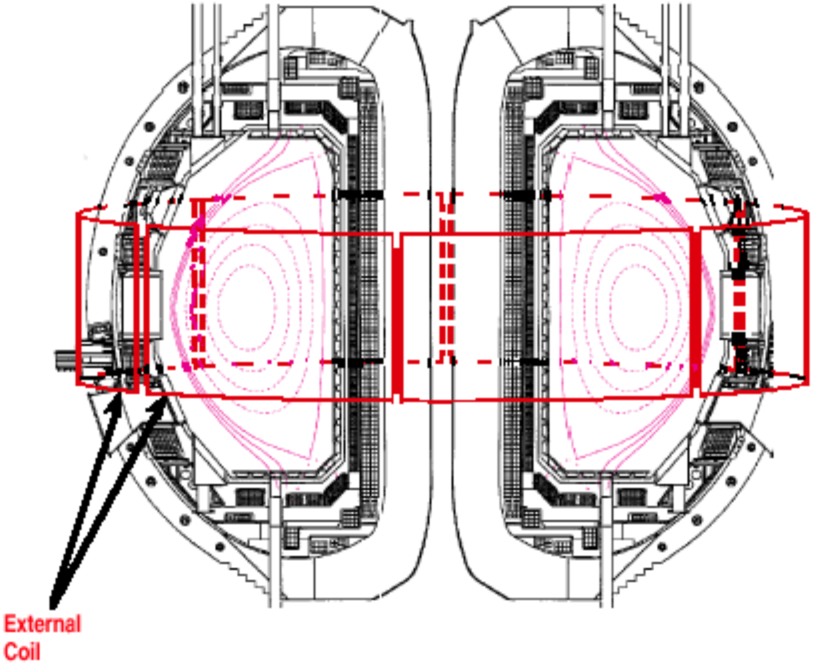


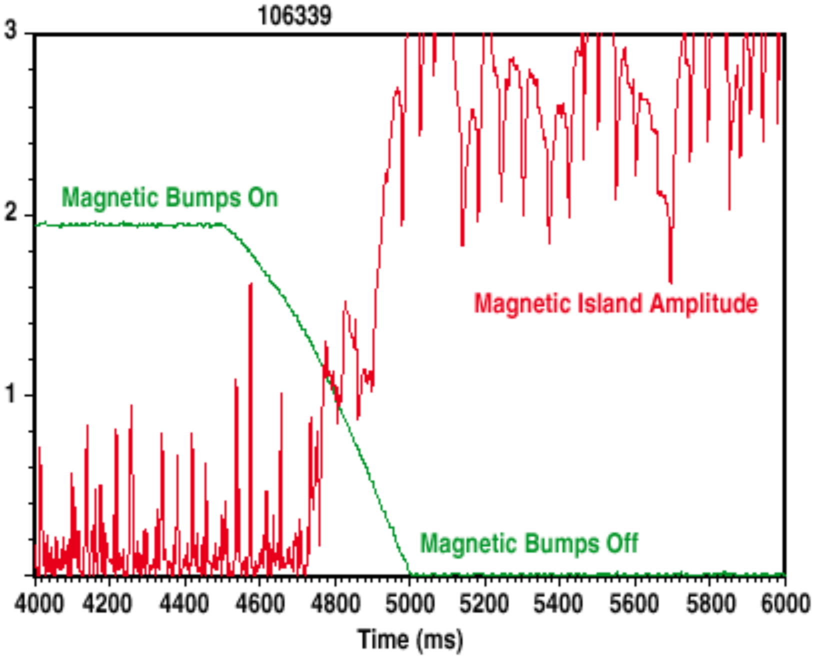
Real-time control of power to avoid machine limits



- Power reduced as soon as mode detected
- RFA Temp>60^{oC} would stop the shot

NON-RESONANT FIELD EXPTS ON D_III_D





NTM stabilized by non-resonant helical fields

Non-resonant helical field stabilization of NTM

- Experimental observation : NTMs of **more than one helicity do not exist simultaneously in an experiment** e.g. if (2,1) grows then (3,2) starts decaying rapidly - so one dominant helicity.

- This suggests that magnetic perturbation of NTM affects stability of another NTM by altering its pressure perturbation.

- Theoretical studies of **mode -mode interaction between NTMs of different helicities** confirmed this idea.

- Can one induce stabilization by using external perturbations of a different helicity? - **YES**

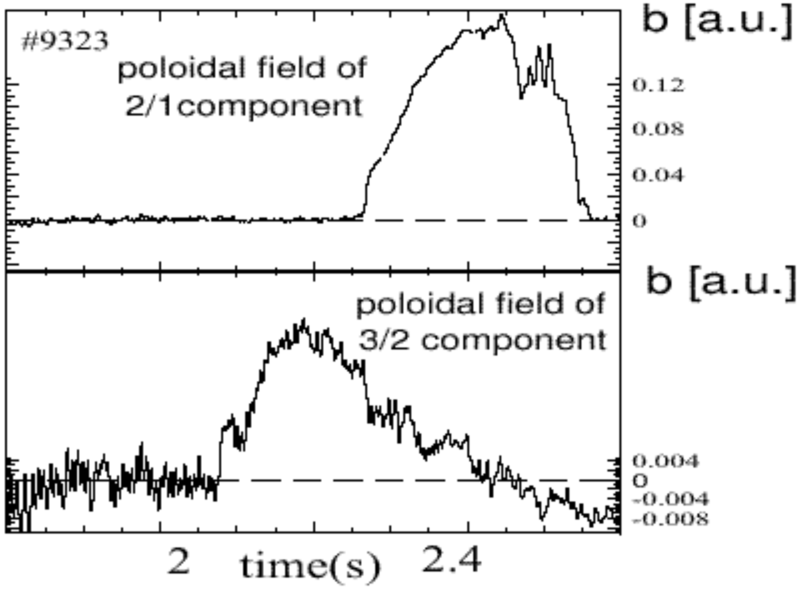


Fig.1 The time evolution of the 2/1 and 3/2 poloidal field amplitude, measured by Mirnov coils.

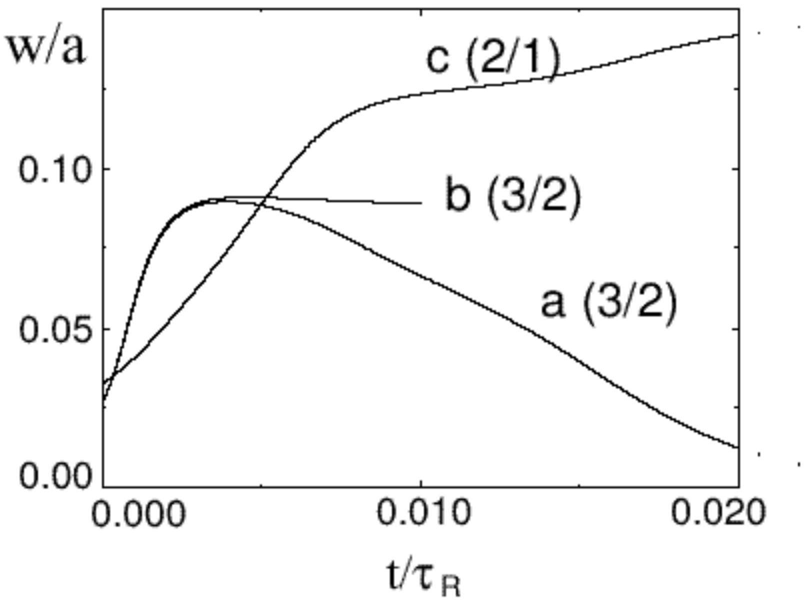


Fig.2 The time evolution of the island width of a 3/2 NTM with (curve a) and without (curve b) a 2/1 mode (curve c).

Comparison of ASDEX data with theoretical modeling of mode interactions
[Qu, Gunter, Lackner]

Effect of static external field (m,n) =(1,3) - theoretical

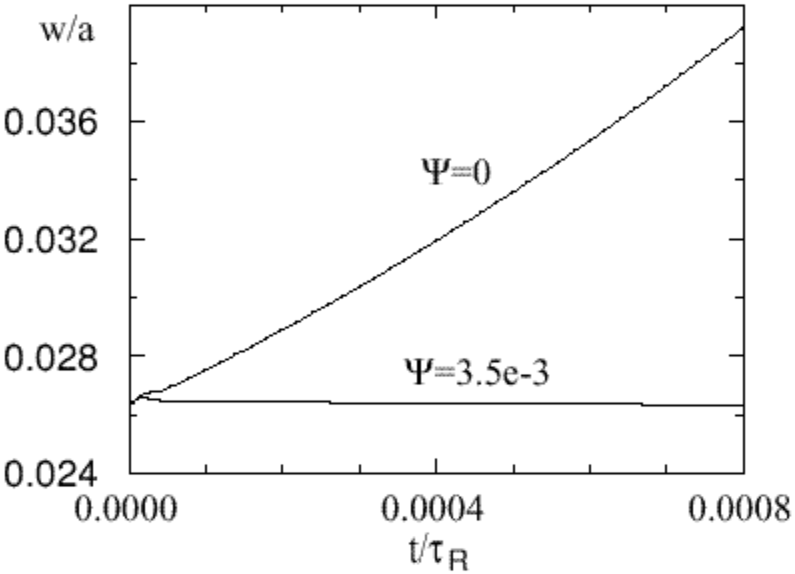


Fig.3 The time evolution of the normalized 3/2 island width with and without the m/n=1/3 helical field.

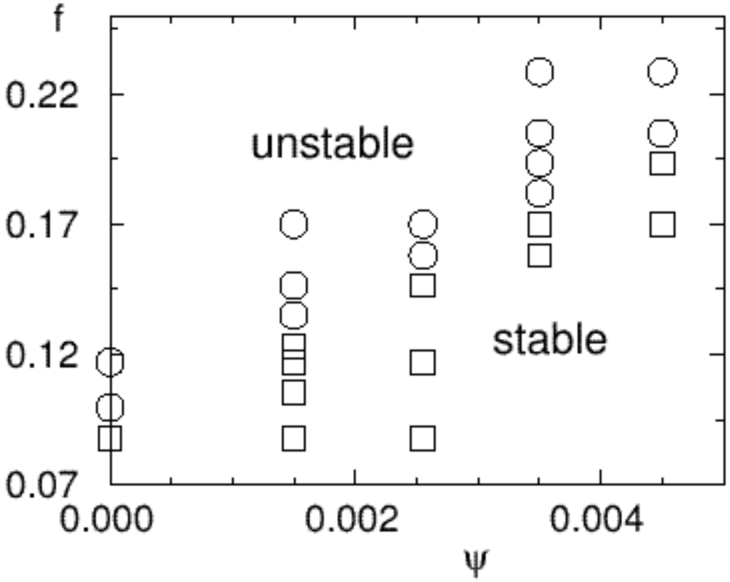


Fig.4 The required ψ for stabilizing the 3/2 mode. The squares (circles) denote the stable (unstable) cases.

$\Psi \equiv \psi_{1/3}(a)/a|\mathbf{B}_0|$ $\quad$ $f \equiv j_{BS}/j_0$

Outstanding Theoretical and Experimental Issues

- **Island width threshold**
  - perpendicular heat transport - local model - improvements necessary - active ongoing theoretical effort
  - neoclassical/ion polarization effects - several open theoretical questions (role of drift waves, ion viscosity effects at high temp, the exact value of the mode frequency, role of energetic ions etc.) - experimental determination also a challenge.

•Seed Island formation

- `standard' NTM initiated by outside MHD event - proper modeling necessary

- 'seedless' NTMs have been seen on TFTR/MAST

  - coupling to an ideal perturbed mode

  - $\Delta' > 0$ modes nonlinearly saturating at small levels?

  - Small scale islands modulated by ion population?

  - turbulence induced trigger

- **Local Current Drive stabilization**
  - works well when island O point is hit - optimization methods being worked out.

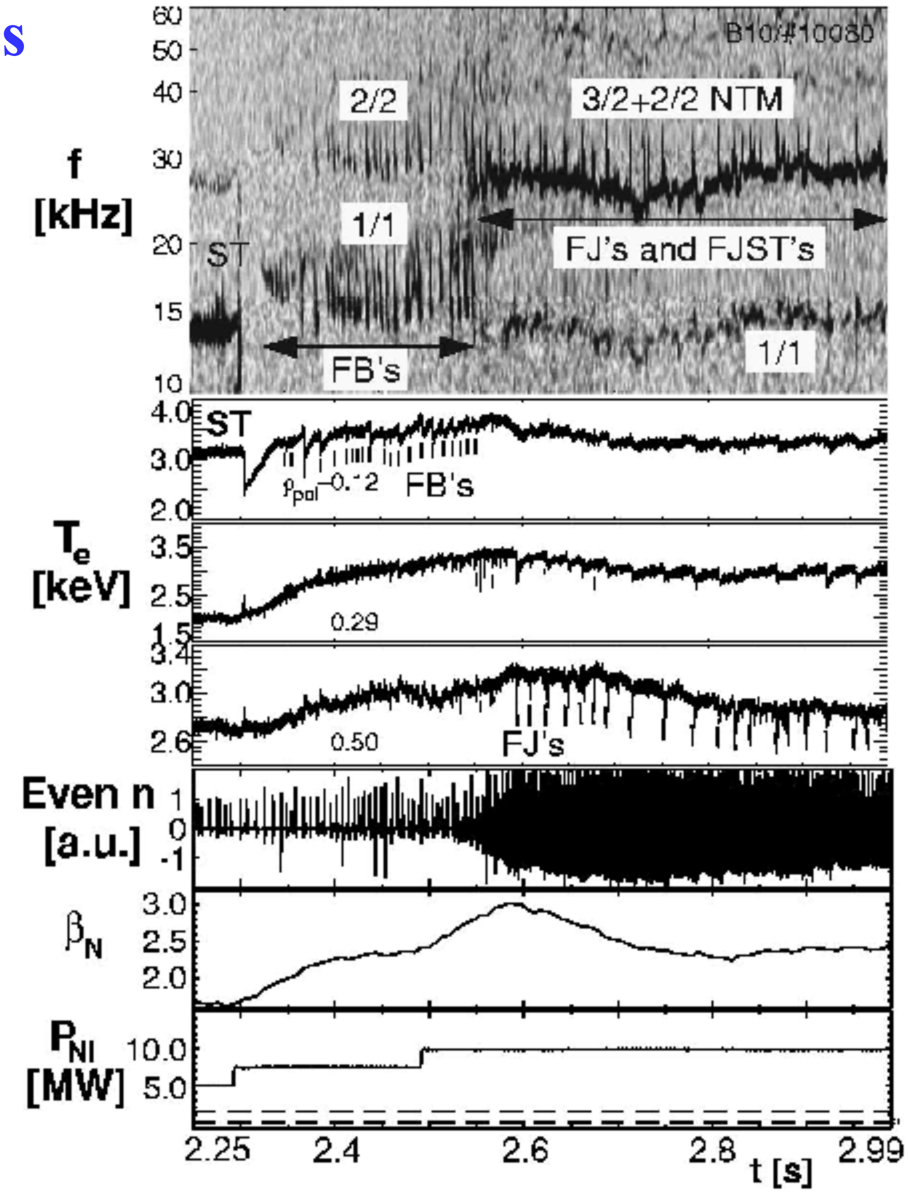
- **Non-resonant Helical perturbation**
  - works well experimentally but mechanism not well understood theoretically
  - slows down rotation - affects other modes e.g. resistive wall mode

- **Interaction of fast particles with NTMs** – open problem

- **Plasma Rotation Effects on NTM** - open problem

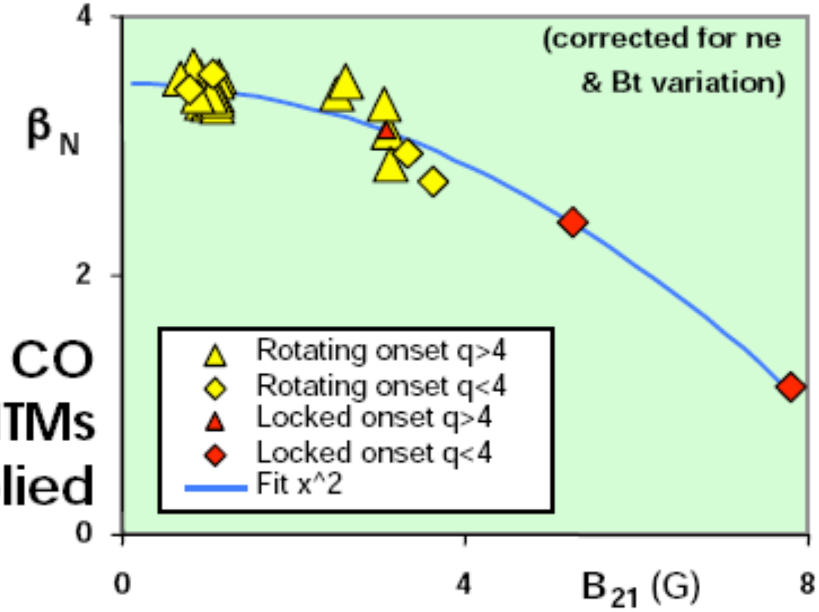
Interaction of fast particles with NTMs lead to FJs in ASDEX U

FB = fishbones
ST = sawteeth
FJ = frequency jump events
FJST = FJ with ST character

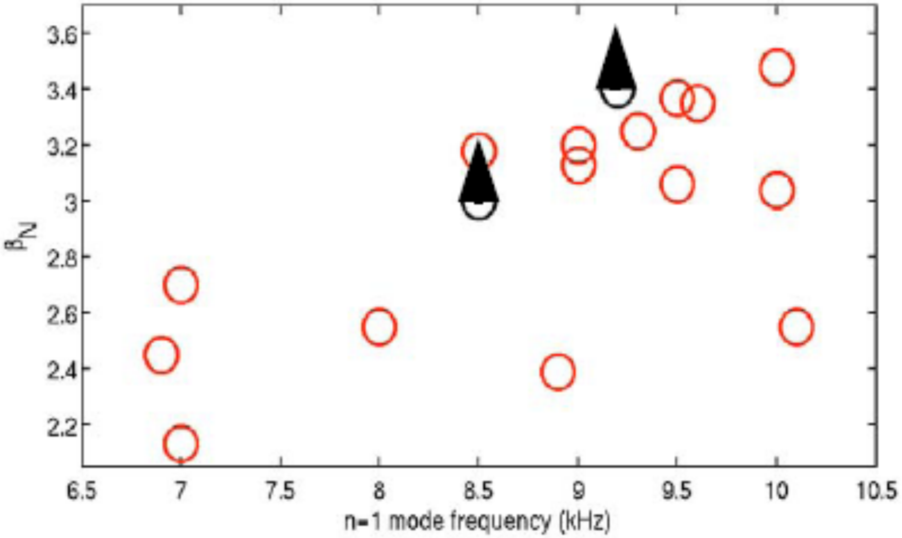


Experimental evidence of flow effects on NTM onset

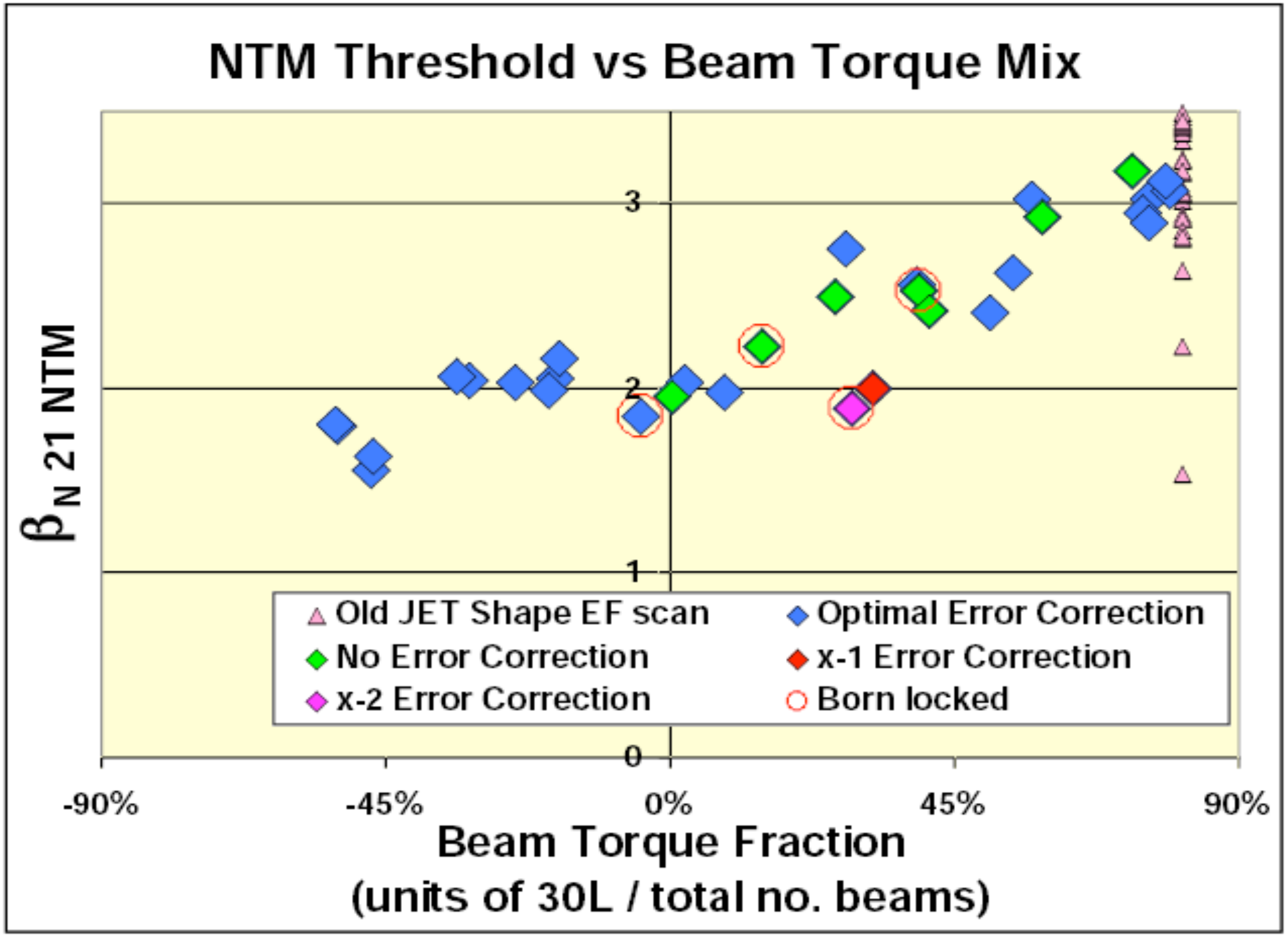
ALL CO
DIII–D 2/1 NTMs
with EF applied



JET 3/2 NTMs: NB momentum scan

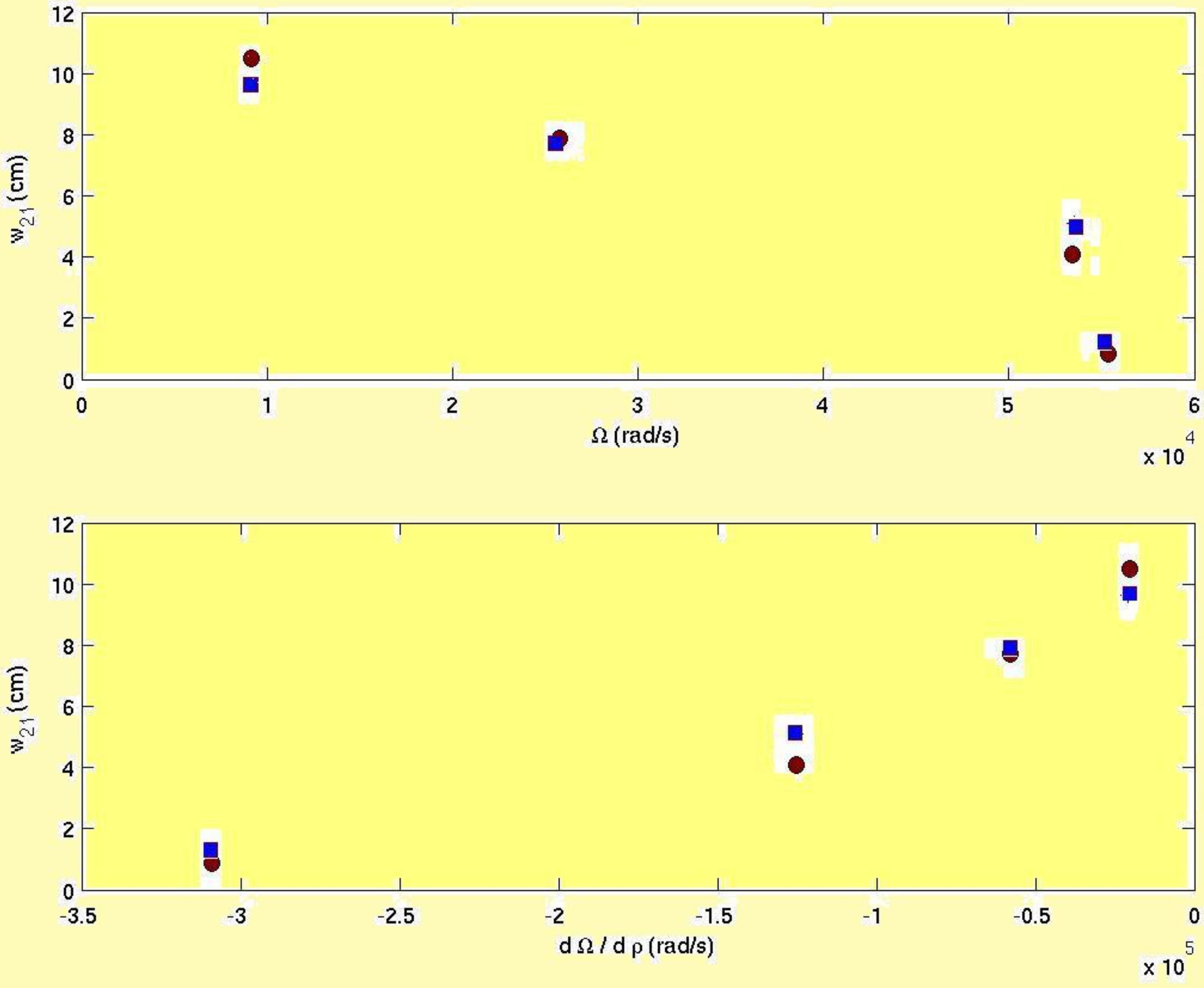


β ramps at fixed co:counter ratio



- **Clear trend towards lower 2/1 NTM β threshold as rotation balances**
  - Suggests thresholds may be lower in ITER

Flow and flow shear effects on saturated island size



Modified Rutherford Equation for NTMs

Pressure/curvature **Neoclassical current**

$$0.41\frac{\partial W}{\partial t} = D_R^{neo}\left[\frac{\Delta'_c}{4} - \frac{19.5}{W}\frac{\epsilon L_s^2}{B_0^2}\frac{\partial p(0)}{\partial \psi} + 0.58\frac{\sqrt{\epsilon}\beta_\theta \frac{L_q}{L_p}}{W}\frac{W^2}{W^2+W_\chi^2}\right.$$

$$\left. + \frac{L_s^2}{k_\theta^2 v_A^2}\left(2.3\frac{(\omega-\omega_E)(\omega-\omega_E-\omega_*)}{W^3} + 0.24\frac{{\omega'_E}^2}{W}\right) - 0.77\frac{L_s}{k_\theta v_A}\frac{\bar{v}_{||0}}{v_A}\frac{\omega'_E}{W}\right]$$

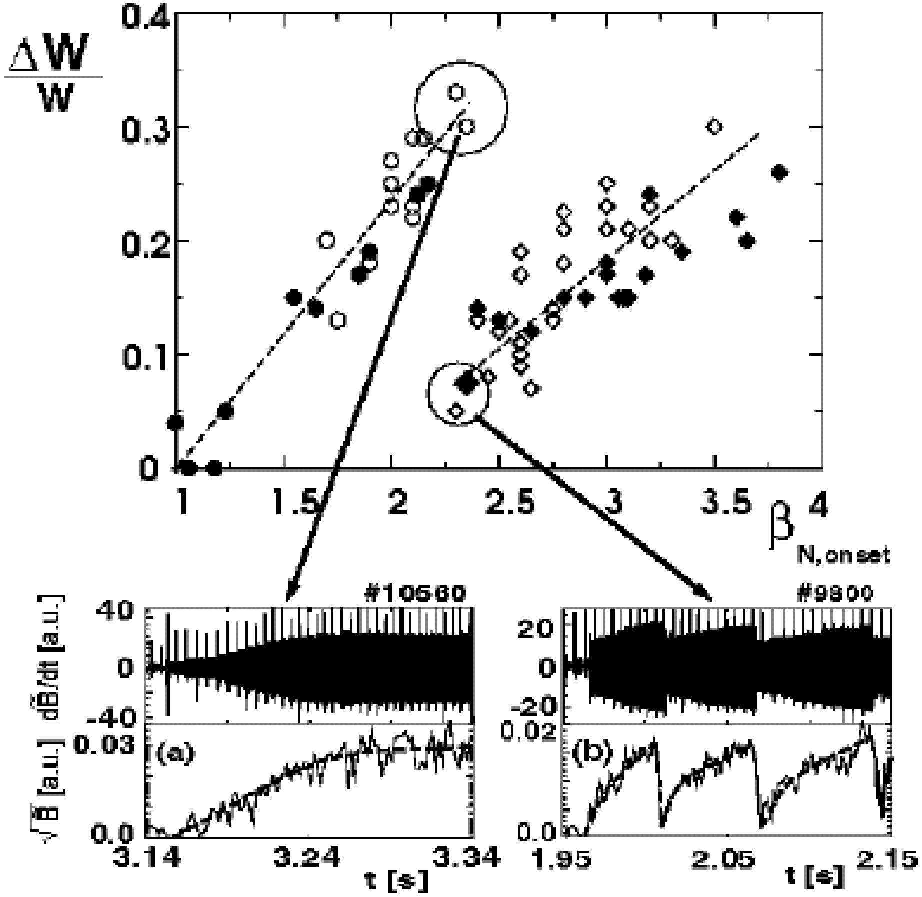
differential flow **flow shear**

polarization current

New NTM regime – Frequently Interrupted Regime

- Happens at higher $\beta_N > 2.3$
- Growth of the NTM is often interrupted by drops in amplitude
- Observed for (3,2) modes in AUG and JET
- Confinement degradation is markedly reduced – so a benign regime
- Possible mechanism – nonlinear coupling between (3,2) NTM, (1,1) and (4,3) mode.

Frequently Interrupted Regime of NTMs



Concluding Remarks

- NTMs are large size magnetic islands driven by neoclassical effects

- Basic physics fairly well understood - modified Rutherford eqn.

- Can have a major impact on tokamak performance by **limiting β**

- Experimentally widely observed in several tokamaks

- **ECCD method of stabilization works** well and is understood

- Still many experimental features (seed island, FJs, non-resonant stabilization etc.) are not well understood.

- **Active area of research offering opportunities for theoretical and experimental insight into reconnection and MHD control issues.**

Some useful references

- O. Sauter et al, Phys Plasmas **4** (1997) 1654
- C.C. Hegna, Phys Plasmas **5** (1998) 1767
- ITER Physics Basis , Nucl. Fusion **47** (2007) Chapter 3 section 2.2